A generalization of Lepage forms in mechanics

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Abstract. In this paper we generalize the concept of a Lepage form, introduced by Krupka, to forms of arbitrary degree in mechanics. These forms allow us to find a suitable representation of the classes of forms, appearing in variational sequences in mechanics. The structure of Lepage 2-forms is discussed in detail. The Lepage equivalents of the dynamical forms are mentioned.
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1. INTRODUCTION

In this paper, a construction is introduced, allowing us to generalize the concept of a Lepage form (Krupka [9, 10]) to forms of arbitrary degree in the higher order variational sequences on fibered manifolds over one-dimensional bases (i.e., in mechanics).

The r-th order variational sequence is by definition the quotient sequence of the De Rham sequence on the r-jet prolongation of a fibered manifold, factored through its contact subsequence (Krupka [12]). Basic general properties of the sequence, and in particular, of the variational terms (lagrangians, Euler-Lagrange forms and Helmholtz-Sonin forms) have been studied by several authors. A complete local representation of the r-th order variational sequence in mechanics was found by Štefánek [19]. Another representation of all classes in the first order variational sequence was given by Krupka [11]. Musilová and Krbek [16] found a representation of the variational terms in higher order variational sequence in mechanics. Kašparová [6] found a representation of classes of n-forms, $(n+1)$-forms and $(n+2)$-forms of the variational sequence in the first order field theory. Her results were extended to the general order by Krbek, Musilová and Kašparová [8]. The representation of all terms in the r-th order field theory were found by Krbek, Musilová [7] by the use of a finite version of Anderson's interior Euler operator [1].

Francaviglia, Palese and Vitolo discussed, among others, such questions as the correspondence of variational sequences and bicomplexes, and their relations to spectral sequences ([4, 5, 20]).

The need of global concepts in higher order variational theory led to the introduction of the so called Lepage n-forms in field theory, and Lepage equivalents of lagrangians. The main idea, going back to Lepage and Dedecker, was that there should exist a

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close connection between the Euler-Lagrange mapping and the exterior derivative of forms (Krupka [10]). Later, the concept of the Lepage form was extended to 2-forms in mechanics and to $(n+1)$-forms in field theory (Krupková [14, 13]); the Lepage forms have been introduced as closed counterparts of the Euler-Lagrange forms. In [14], the Lepage forms have been applied to the inverse problem in higher order mechanics, and to the order reducibility problem.

In our generalization of Lepage forms we use a slight (finite order) modification of an operator $\mathscr{I}$, acting on forms on jet manifolds, given by Anderson [1] for the case of the variational bicomplex and called by Anderson the interior Euler operator. This operator was already used, and denoted by different symbols, by Kuperschmidt [15], Dedecker and Tulczyjew [3], and Bauderon [2].

2. VARIATIONAL SEQUENCE

Let $\pi : Y \to X$ be a fibered manifold with fibered coordinate systems (V,ψ), $\psi = (t,q^\sigma)$, on Y and (U,φ), $\varphi = (t)$ on X, $\dim X = 1$, $\dim Y = m+1$. Denote by $\pi^r : J^rY \to X$ or just J^rY the r-jet prolongation of the fibered manifold $\pi : Y \to X$, the coordinate system is (V^r,ψ^r), $\psi^r = (t,q^\sigma,q^\sigma_1,\cdots,q^\sigma_r)$ on J^rY. For small r we denote $q^\sigma_0 = q^\sigma$, $q^\sigma_1 = \dot{q}^\sigma$, $q^\sigma_2 = \ddot{q}^\sigma$.The canonical jet projections are $\pi^{r,s} : J^rY \to J^sY$, $r > s$ and $\pi^{r,0} : J^rY \to Y$.

A differential k-form ρ on J^rY is called *contact*, if it vanishes along the r-jet prolongation $J^r\gamma$ of every section γ of π.

If (V,ψ), $\psi = (t,q^\sigma)$, is a fibered chart on Y, then we often use the *contact basis* $dt,\omega^\sigma,\omega^\sigma_1,\ldots,\omega^\sigma_r,dq^\sigma_{r+1}$ on $V^{r+1} = (\pi^{r+1,0})^{-1}V$ given by the forms

$$\omega^\sigma_j = dq^\sigma_j - q^\sigma_{j+1}dt, \qquad 0 \le j \le r. \tag{1}$$

Recall that a form which contains exactly k expressions (1) is called k-contact. Every form ρ on J^rY can be uniquely decomposed, after the lifting to $J^{r+1}Y$, as the sum of the k-contact components $p_k\rho$.

Let Ω^r_k be the direct image of the sheaf of smooth k-forms over J^rY by the jet projection $\pi^{r,0}$, where $k \ge 0$. Denote

$$\Omega^r_{0,c} = \{0\}, \quad \Omega^r_{k,c} = \ker p_{k-1}, \quad \Theta^r_k = \Omega^r_{k,c} + d\Omega^r_{k-1,c}, \tag{2}$$

where $k \ge 1$, and $d\Omega^r_{k-1,c}$ is the image sheaf of $\Omega^r_{k-1,c}$ by d. Then for every open set $V \subset Y$, Ω^r_kV (resp. $\Omega^r_{k,c}V$) is the Abelian group of k-forms (resp. k-contact k-forms) on $V^r = (\pi^{r,0})^{-1}(V)$, $d\Omega^r_{k-1,c}V$ is the Abelian group of forms which can be locally expressed as differentials of $(k-1)$-contact $(k-1)$-forms on V^r, and Θ^r_kV is a subgroup of Ω^r_kV. We get a sequence

$$0 \to \Theta^r_1 \to \Theta^r_2 \to \Theta^r_3 \to \ldots \to \Theta^r_M \to 0, \tag{3}$$

in which all arrows denote the exterior differentiation d, and $M = mr+1$. Sequence (3) is a subsequence of the De Rham sequence

$$0 \to \mathbb{R}_Y \to \Omega^r_0 \to \Omega^r_1 \to \Omega^r_2 \to \ldots \to \Omega^r_{N-1} \to \Omega^r_N \to 0, \tag{4}$$

where $N = \dim J^rY = 1 + m(r+1)$. The quotient sequence

$$\begin{aligned} 0 \to \mathbb{R}_Y \to \Omega_0^r \to \Omega_1^r/\Theta_1^r \to \Omega_2^r/\Theta_2^r \to \dots \\ \dots \to \Omega_M^r/\Theta_M^r \to \Omega_{M+1}^r \to \dots \to \Omega_{N-1}^r \to \Omega_N^r \to 0 \end{aligned} \tag{5}$$

is also exact. Sequence (5) is called the *r-th order variational sequence*. The class of a differential form $\rho \in \Omega_k^r V$ in the variational sequence (5) is denoted by $[\rho]$.

The quotient mapping $E : \Omega_k^r/\Theta_k^r \to \Omega_{k+1}^r/\Theta_{k+1}^r$ is defined by

$$E([\rho]) = [d\rho]. \tag{6}$$

This mapping satisfies the condition $E^2 = 0$. The quotient mapping $E : \Omega_1^r/\Theta_1^r \to \Omega_2^r/\Theta_2^r$ is called the *Euler-Lagrange mapping*. The quotient mapping $E : \Omega_2^r/\Theta_2^r \to \Omega_3^r/\Theta_3^r$ is called the *Helmholtz-Sonin mapping*.

A *lagrangian* of order r is a π^r-horizontal n-form λ. In coordinates, the following can be written

$$\lambda = L dt, \tag{7}$$

where L is a function on J^rY called *Lagrange function*.

Let ρ be a 1-form on J^rY. A form ρ is called a *Lepage* 1-*form* if $p_1 d\rho$ is a $\pi^{r+1,0}$-horizontal 2-form. A Lepage form ρ is called a *Lepage equivalent* of a lagrangian λ if $h\rho = \lambda$. It is known that in higher order mechanics, Lepage equivalents are uniquely determined by lagrangians. We denote by θ_λ the Lepage equivalent of a lagrangian λ. If $r = 1$, θ_λ is the well known *Poincaré-Cartan form*, if $r > 1$, we have the *generalized Poincaré-Cartan form*. If in a fibered chart $\lambda = L\,dt$, then

$$p_1 d\theta_\lambda = E_\sigma(L)\omega^\sigma \wedge dt, \tag{8}$$

where

$$E_\sigma(L) = \sum_{l=0}^{r} (-1)^l \frac{d^l}{dt^l} \frac{\partial L}{\partial q_l^\sigma}. \tag{9}$$

The form (8) is called the *Euler-Lagrange form* and it is denoted by E_λ. The components (9) are called the *Euler-Lagrange expressions*.

3. THE INTERIOR EULER-LAGRANGE OPERATOR

We recall basic properties of the interior Euler-Lagrange operator rewritten in the form presented in Šeděnková [18].

Let (V, ψ), $\psi = (t, q^\sigma)$, be a fibered chart on Y and let (V^{2r+1}, ψ^{2r+1}), $\psi^{2r+1} = (t, q^\sigma, q_1^\sigma, \dots, q_{2r+1}^\sigma)$, be the associated fibered chart on $J^{2r+1}Y$. We set

$$\Xi = \frac{\partial}{\partial t} + \sum_{j=0}^{2r} q_{j+1}^\sigma \frac{\partial}{\partial q_j^\sigma}, \tag{10}$$

Ξ is a vector field on V^{2r+1}. If $\rho \in \Omega^r_{k+1}V$, $k \geq 1$, we define a form on V^{2r+1} by

$$\mathscr{I}_{(V,\psi)}(\rho) = \frac{1}{k}\omega^\alpha \wedge \sum_{j=0}^{r} (-1)^j \partial^j_\Xi i_{\frac{\partial}{\partial q^\alpha_j}} p_k\rho, \tag{11}$$

where ∂_Ξ is the Lie derivative with respect to the vector field Ξ, ∂^j_Ξ is the j-th power of ∂_Ξ, and $i_{\partial/\partial q^\alpha_j}$ denotes the contraction by the vector field $\partial/\partial q^\alpha_j$. For $k = 0$ and $\rho \in \Omega^r_1 V$, we define

$$\mathscr{I}_{(V,\psi)}(\rho) = h\rho. \tag{12}$$

Note that the form $\mathscr{I}_{(V,\psi)}(\rho)$ depends only on the k-contact $(k+1)$-form $p_k\rho$.

The following two lemmas and Theorem 1 can be proved in fibered coordinates.

Lemma 1. *Let $\rho \in \Omega^r_{k+1}V$, $k \geq 1$. Then the following equation is satisfied*

$$\frac{1}{k}\omega^\alpha \wedge \sum_{j=0}^{r} (-1)^j \partial^j_\Xi i_{\frac{\partial}{\partial q^\alpha_j}} p_k\rho = p_k\rho + \frac{1}{k}\sum_{j=1}^{r}\sum_{l=1}^{j} (-1)^l \binom{j}{l} \partial^l_\Xi (\omega^\alpha_{j-l} \wedge i_{\frac{\partial}{\partial q^\alpha_j}} p_k\rho). \tag{13}$$

Lemma 2. *Let (V,ψ), $\psi = (t, q^\sigma)$, $(\bar{V}, \bar{\psi})$, $\bar{\psi} = (\bar{t}, \bar{q}^\sigma)$ be two fibered charts on Y such that $V \cap \bar{V} \neq 0$. Then for every $\rho \in \Omega^r_{k+1}(V \cap \bar{V})$, $k \geq 0$,*

$$\mathscr{I}_{(V,\psi)}(\rho) = \mathscr{I}_{(\bar{V},\bar{\psi})}(\rho). \tag{14}$$

It follows from Lemma 2 that equations (11) define a global operator $\mathscr{I} : \Omega^r_{k+1} \to \Omega^{2r+1}_{k+1}$. $\mathscr{I}$ is called the *interior Euler-Lagrange operator*. The differential form $\mathscr{I}(\rho)$ is called the *canonical representative* of a differential form ρ. The operator $\mathscr{I}$ generates new sequence

$$0 \to \mathbb{R}_Y \to \Omega^r_0 \to \mathscr{I}\Omega^r_1 \to \mathscr{I}\Omega^r_2 \to \ldots \to \mathscr{I}\Omega^r_{N-1} \to \mathscr{I}\Omega^r_N \to 0, \tag{15}$$

which is isomorfic with the variational sequence (5).

The following theorem characterizes properties of $\mathscr{I}$. In particular, it turns out that the kernels of $\mathscr{I}$ coincide with the spaces in the subsequence (3) of the De Rham sequence (4).

Theorem 1. *Let $\pi : Y \to X$ be a fibered manifold over one-dimensional base X. Let $k \geq 0$.*

(a) *For every open set $V \subset Y$ and every $\rho \in \Omega^r_{k+1}V$, $\mathscr{I}(\rho)$ lies in the same class as $(\pi^{2r+1,r})^*\rho$.*
(b) *The operator $\mathscr{I}$ satisfies $\mathscr{I}^2 = \mathscr{I}$ (up to the canonical projection).*
(c) *For every open set $V \subset Y$, the kernels of $\mathscr{I}$ coincide with $\Theta^r_{k+1}V$.*

4. LEPAGE FORMS

Let $k \geq 0$. A form $\rho \in \Omega^r_{k+1} V$ is called a *Lepage form*, if

$$p_{k+1} d\rho = \mathscr{I}(d\rho). \tag{16}$$

For $k = 0$, this definition reduces to the original one (Krupka [9, 10]; for more details we refer to Šeděnková [18]). If ρ is a Lepage form, then the forms $d\rho$ and $\rho + d\eta$, where η is arbitrary, are trivially also Lepage forms. The meaning of Lepage forms consists in a generalization of formulas (8), (9); if $k = 1$, then $p_2 d\rho$ is the *Helmholtz-Sonin form* (compare with Krupka [11] for the first order case).

We now analyze the structure of Lepage 2-forms in higher order mechanics. Because of the lack of space, we restrict ourselves to preliminary results; more details as well as proofs will be given elsewhere.

Theorem 2. *Let $\rho \in \Omega^1_2 V$, let in a fibered chart*

$$\rho = a_\sigma \omega^\sigma \wedge dt + b_\sigma d\dot{q}^\sigma \wedge dt + c_{\sigma\nu} \omega^\sigma \wedge \omega^\nu + d_{\sigma\nu} d\dot{q}^\sigma \wedge \omega^\nu + e_{\sigma\nu} d\dot{q}^\sigma \wedge d\dot{q}^\nu, \tag{17}$$

the coefficients $c_{\sigma\nu}$, $e_{\sigma\nu}$ are antisymmetric in σ, ν. The following three conditions are equivalent:

(a) *ρ is a Lepage form*

(b) *ρ satisfies*

$$\frac{\partial e_{\sigma\nu}}{\partial \dot{q}^\lambda} + \frac{\partial e_{\nu\lambda}}{\partial \dot{q}^\sigma} + \frac{\partial e_{\lambda\sigma}}{\partial \dot{q}^\nu} = 0, \tag{18}$$

$$d_{\sigma\nu} - d_{\nu\sigma} - \frac{\partial b_\sigma}{\partial \dot{q}^\nu} + \frac{\partial b_\nu}{\partial \dot{q}^\sigma} + 2\frac{\partial e_{\sigma\nu}}{\partial t} + 2\frac{\partial e_{\sigma\nu}}{\partial q^\lambda}\dot{q}^\lambda = 0, \tag{19}$$

$$\frac{\partial d_{\sigma\nu}}{\partial \dot{q}^\lambda} - \frac{\partial d_{\nu\sigma}}{\partial \dot{q}^\lambda} + \frac{\partial d_{\lambda\sigma}}{\partial \dot{q}^\nu} - \frac{\partial d_{\lambda\nu}}{\partial \dot{q}^\sigma} + 2\frac{\partial e_{\lambda\sigma}}{\partial q^\nu} - 2\frac{\partial e_{\lambda\nu}}{\partial q^\sigma} = 0, \tag{20}$$

$$\frac{\partial a_\nu}{\partial \dot{q}^\sigma} - \frac{\partial a_\sigma}{\partial \dot{q}^\nu} + \frac{\partial b_\nu}{\partial q^\sigma} - \frac{\partial b_\sigma}{\partial q^\nu} + \frac{\partial d_{\sigma\nu}}{\partial t} - \frac{\partial d_{\nu\sigma}}{\partial t} + \frac{\partial d_{\sigma\nu}}{\partial q^\lambda}\dot{q}^\lambda - \frac{\partial d_{\nu\sigma}}{\partial q^\lambda}\dot{q}^\lambda + 4c_{\sigma\nu} = 0. \tag{21}$$

(c) *There exist functions A_σ, and a 1-form η such that*

$$\rho = A_\sigma \omega^\sigma \wedge dt + \frac{1}{4}\left(\frac{\partial A_\sigma}{\partial \dot{q}^\nu} - \frac{\partial A_\nu}{\partial \dot{q}^\sigma}\right)\omega^\sigma \wedge \omega^\nu + d\eta. \tag{22}$$

Now we consider second order Lepage 2-forms. We have the following result.

Theorem 3. *Let $\rho \in \Omega^2_2 V$, let in a fibered chart*

$$\begin{aligned} \rho &= a_\sigma \omega^\sigma \wedge dt + b_\sigma \dot{\omega}^\sigma \wedge dt + c_\sigma d\ddot{q}^\sigma \wedge dt + d_{\sigma\nu}\omega^\sigma \wedge \omega^\nu + e_{\sigma\nu}\dot{\omega}^\sigma \wedge \omega^\nu \\ &+ f_{\sigma\nu}\dot{\omega}^\sigma \wedge \dot{\omega}^\nu + g_{\sigma\nu} d\ddot{q}^\sigma \wedge \omega^\nu + h_{\sigma\nu} d\ddot{q}^\sigma \wedge \dot{\omega}^\nu + i_{\sigma\nu} d\ddot{q}^\sigma \wedge d\ddot{q}^\nu \end{aligned} \tag{23}$$

and

$$
\begin{aligned}
p_2 d\rho &= P_{\sigma\nu}\omega^\sigma\wedge\omega^\nu\wedge dt + Q_{\sigma\nu}\dot\omega^\sigma\wedge\omega^\nu\wedge dt + R_{\sigma\nu}\ddot\omega^\sigma\wedge\omega^\nu\wedge dt \\
&+ S_{\sigma\nu}\dot\omega^\sigma\wedge\dot\omega^\nu\wedge dt + T_{\sigma\nu}\ddot\omega^\sigma\wedge\dot\omega^\nu\wedge dt + U_{\sigma\nu}\ddot\omega^\sigma\wedge\ddot\omega^\nu\wedge dt,
\end{aligned}
\tag{24}
$$

the coefficients $d_{\sigma\nu}$, $f_{\sigma\nu}$, $i_{\sigma\nu}$, $P_{\sigma\nu}$, $S_{\sigma\nu}$, $U_{\sigma\nu}$ are antisymmetric in σ,ν. The following three conditions are equivalent:

(a) *ρ is a Lepage form*

(b) *The components of $p_2 d\rho$ satisfy*

$$
\begin{aligned}
U_{\sigma\nu} - U_{\nu\sigma} = 0, \qquad T_{\sigma\nu} = 0, \qquad S_{\sigma\nu} - S_{\nu\sigma} = 0, \\
R_{\sigma\nu} + R_{\nu\sigma} = 0, \qquad Q_{\sigma\nu} - Q_{\nu\sigma} - 2\tfrac{d}{dt}R_{\sigma\nu} = 0.
\end{aligned}
\tag{25}
$$

(c) *There exist functions A_σ satisfying*

$$
\frac{\partial}{\partial\ddot q^\tau}\left(\frac{\partial a_\sigma}{\partial\ddot q^\nu} - \frac{\partial a_\nu}{\partial\ddot q^\sigma}\right) = 0 \tag{26}
$$

and a 1-forn η such that

$$
\begin{aligned}
\rho &= a_\sigma\omega^\sigma\wedge dt + \frac14\left(\frac{\partial a_\sigma}{\partial\dot q^\nu} - \frac{\partial a_\nu}{\partial\dot q^\sigma} - \frac{d}{dt}\left(\frac{\partial a_\sigma}{\partial\ddot q^\nu} - \frac{\partial a_\nu}{\partial\ddot q^\sigma}\right)\right)\omega^\sigma\wedge\omega^\nu \\
&- \frac12\left(\frac{\partial a_\sigma}{\partial\ddot q^\nu} + \frac{\partial a_\nu}{\partial\ddot q^\sigma}\right)\dot\omega^\sigma\wedge\omega^\nu + d\eta,
\end{aligned}
\tag{27}
$$

We note that the coefficients in (24) can be expressed in terms of the coefficients in (23); then (25) become conditions for the coefficients of ρ.

Analogous results can also be given for r-th order 2-forms.

Theorem 4. *Let $\rho\in\Omega_2^r V$, let in a fibered chart*

$$
\begin{aligned}
\rho &= \sum_{i=0}^{r-1} a_\sigma^i\omega_i^\sigma\wedge dt + b_\sigma^r dq_r^\sigma\wedge dt \\
&+ \sum_{i,j=0}^{r-1} c_{\sigma\nu}^{\,i\,j}\omega_i^\sigma\wedge\omega_j^\nu + \sum_{j=0}^{r-1} d_{\sigma\nu}^{\,r\,j} dq_r^\sigma\wedge\omega_j^\nu + e_{\sigma\nu}^{\,r\,r} dq_r^\sigma\wedge dq_r^\nu
\end{aligned}
\tag{28}
$$

and

$$
p_2 d\rho = \sum_{j=0}^{r} H_{\sigma\nu}^{0\,j}\omega^\sigma\wedge\omega_j^\nu\wedge dt + \sum_{i,j=1}^{r} H_{\sigma\nu}^{\,i\,j}\omega_i^\sigma\wedge\omega_j^\nu\wedge dt. \tag{29}
$$

the coefficients $e_{\sigma\nu}^{\,r\,r}$ are antisymmetric in σ,ν, the coefficients $c_{\sigma\nu}^{\,i\,j}$, $H_{\sigma\nu}^{\,i\,j}$ are antisymmetric in pairs $\binom{i}{\sigma}$, $\binom{j}{\nu}$. Then ρ is Lepage if and only if

$$
\begin{aligned}
H_{\sigma\nu}^{\,i\,j} - H_{\nu\sigma}^{\,j\,i} &= 0, \qquad 1\le i,j\le r, \\
H_{\sigma\nu}^{0\,j} + (-1)^j H_{\nu\sigma}^{0\,j} + \textstyle\sum_{l=j+1}^{r}(-1)^l\binom{l}{j}\frac{d^{l-j}}{dt^{l-j}}H_{\nu\sigma}^{0\,l} &= 0, \qquad 1\le j\le r.
\end{aligned}
\tag{30}
$$

Finally, we define Lepage equivalents of the canonical representatives of differential forms. Let $\beta \in \Omega^s_{k+1}/\Theta^s_{k+1}$ be a class, i.e., let $\beta = \mathscr{I}(\eta)$ for some $\eta \in \Omega^s_{k+1}V$. A form $\rho \in \Omega^r_{k+1}V$ is said to be a *Lepage equivalent* of β, if ρ is a Lepage form, and

$$p_k\rho = \beta. \tag{31}$$

In particular, this definition includes Lepage equivalents of *dynamical forms* (i.e., the canonical representatives of 2-forms). In particular, let $E = E_\sigma\omega^\sigma \wedge dt$ be the second order dynamical form with the functions E_σ linear in coordinates $\ddot{q}^\nu$. Then Lepage equivalent ρ_E of the dynamical form E has the form

$$\begin{aligned} \rho_E &= E_\sigma\omega^\sigma \wedge dt + \frac{1}{4}\left(\frac{\partial E_\sigma}{\partial \dot{q}^\nu} - \frac{\partial E_\nu}{\partial \dot{q}^\sigma} - \frac{d}{dt}\left(\frac{\partial E_\sigma}{\partial \ddot{q}^\nu} - \frac{\partial E_\nu}{\partial \ddot{q}^\sigma}\right)\right)\omega^\sigma \wedge \omega^\nu \\ &- \frac{1}{2}\left(\frac{\partial E_\sigma}{\partial \ddot{q}^\nu} + \frac{\partial E_\nu}{\partial \ddot{q}^\sigma}\right)\dot{\omega}^\sigma \wedge \omega^\nu \end{aligned} \tag{32}$$

(compare with second order Lepage form (27)).

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