Control Algorithms with Suppression of Measurable Disturbances: Comparison of Two Methods

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Abstract: - Many processes are affected by external disturbances caused by the variation of variables that can be measured. This paper compares two control strategies which are suitable for rejection of measurable disturbances. The first method which can successfully handle known measurable disturbances is model predictive control (MPC). Known disturbances can be taken explicitly into account in predictive control. Two different approaches to computation of multi–step–ahead predictions incorporating known measurable disturbances into prediction equations are proposed. The second control algorithm is designed using polynomial theory developed for linear controlled systems. Both methods are based on a same model of a controlled process. Simulation results are also included and quality of control achieved by both methods is compared and discussed.

Key-Words: - Predictive control, Polynomial methods, Disturbance rejection, Diophantine equations, Prediction, CARIMA model

1 Introduction

Many processes are affected by external disturbances caused by the variation of variables that can be measured. This situation is typical in processes whose outputs are affected by variations of the load regime. This paper compares two control strategies which are suitable for rejection of measurable disturbances. The first method which can successfully handle known measurable disturbances is model predictive control (MPC) [1], [2], [3]. Theoretical research in the area of predictive control has a great impact on the industrial world and there are many applications of predictive control in industry. Its development has been significantly influenced by industrial practice. At present, predictive control with a number of real industrial applications belongs among the most often implemented modern industrial process control approaches. First predictive control algorithms were implemented in industry as an effective tool for control of multivariable industrial processes with constraints more than twenty five years ago. The use of predictive control was limited on control of namely rather slow processes due to the amount of computation required. At present, with the computing power available today, this is not an essential problem. A fairly actual applications of predictive control are presented in [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14]. An extensive surveys of industrial applications of predictive control are presented in [15], [16], [17].

The second algorithm is designed using polynomial theory developed for linear controlled systems [18], [19]. Both methods are based on a same model of a controlled process.

Incorporation of disturbances to predictive control requires that the disturbance in the future is known. On the other hand a course of the known disturbance can be arbitrary. A controller based on polynomial methods can handle only with disturbances defined by a defined mathematical function from a certain class. In our case a sinusoidal disturbance was chosen in both cases. The proposed controllers then enable disturbance rejection of sinusoidal disturbance signals. This type of disturbance can occur for example in an electrical system where electromagnetic field of AC power lines is superimposed on the electromagnetic field of the control lines.

2 Theoretical Background

2.1 Predictive Control

The term Model Predictive Control designates a class of control methods which have common particular attributes [20], [21]:

- Mathematical model of a systems control is used for prediction of future control of a systems output.
- The input reference trajectory in the future is known.
- A computation of the future control sequence includes minimization of an appropriate objective function (usually quadratic one) with the future trajectories of control increments and control errors.

Only the first element of the control sequence is applied and the whole procedure of the objective function minimization is repeated in the next sampling period.

The principle of Model Predictive Control [22], [23] is shown in Fig. 1, where $u(t)$ is the manipulated variable, $y(t)$ is the process output and $w(t)$ is the reference signal, N_1, N_2 and N_u are called minimum, maximum and control horizon. This principle is possible to define as follows:

1. The process model is used to predict the future outputs over some horizon. The predictions are calculated based on information up to time k and on the future control actions that are to be determined.
2. The future control trajectory is calculated as a solution of an optimisation problem consisting of an objective function and constraints. The cost function comprises future output predictions, future reference trajectory, and future control actions.
3. Although the whole future control trajectory was calculated in the previous step, only first element is actually applied to the process. At the next sampling time the procedure is repeated. This is known as the Receding Horizon concept.

The computation of a control law of MPC is mostly based on minimization of the following criterion.

$$J(k)=\sum_{j=N_1}^{N_2}\boldsymbol{e}(k+j)^2+\lambda\sum_{j=1}^{N_u}\Delta\boldsymbol{u}(k+j)^2 \qquad (1)$$

where $e(k+j)$ is a vector of predicted control errors, $\Delta u(k+j)$ is a vector of future increments of manipulated variables, N_1 and N_2 are minimum and maximum prediction horizons, N_u is length of the control horizon and λ is a weighting factor of control increments.

A predictor in a vector form is given by

$$\hat{\boldsymbol{y}}=\boldsymbol{G}\Delta\boldsymbol{u}+\boldsymbol{y}_0 \qquad (2)$$

where $\hat{y}$ is a vector of systems output predictions along the horizon of the length N_2-N_1. The first element in the equation (2) represents the forced response of the system. $\Delta\boldsymbol{u}$ is a vector of control increments and $\boldsymbol{G}$ is a matrix of the dynamics which contains values of the step sequence. $\boldsymbol{y}_0$ is the free response vector. It is that part of the systems output prediction which is determined by past values of the systems inputs and outputs (the forced response is determined by future values of increments of the manipulated variable).

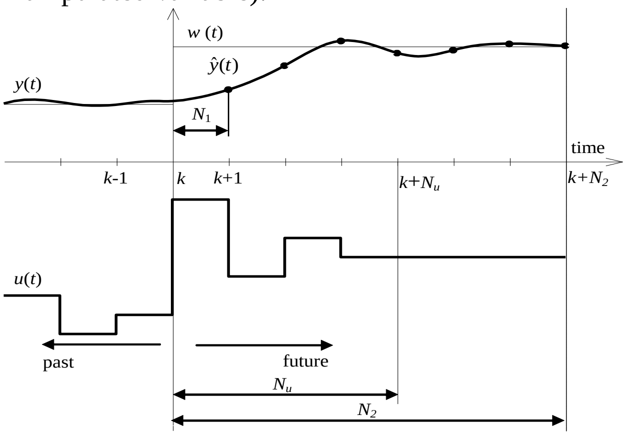

Fig. 1 Principle of predictive control

The cost function (1) can be modified to the form below

$$\begin{aligned}J&=(\hat{\boldsymbol{y}}-\boldsymbol{w})^T(\hat{\boldsymbol{y}}-\boldsymbol{w})+\lambda\Delta\boldsymbol{u}^T\Delta\boldsymbol{u}=\\&=(\boldsymbol{G}\Delta\boldsymbol{u}+\boldsymbol{y}_0-\boldsymbol{w})^T(\boldsymbol{G}\Delta\boldsymbol{u}+\boldsymbol{y}_0-\boldsymbol{w})+\lambda\Delta\boldsymbol{u}^T\Delta\boldsymbol{u}\end{aligned} \qquad (3)$$

Where $\boldsymbol{w}$ is the vector of future reference trajectory. Minimisation of the cost function (3) now becomes a direct problem of linear algebra. The solution in an unconstrained case can be found by setting partial derivative of J with respect to $\Delta\boldsymbol{u}$ as zero and yields

$$\Delta\boldsymbol{u}=(\boldsymbol{G}^T\boldsymbol{G}+\lambda\boldsymbol{I})^{-1}\boldsymbol{G}^T(\boldsymbol{w}-\boldsymbol{y}_0) \qquad (4)$$

where the gradient $\boldsymbol{g}$ and Hessian $\boldsymbol{H}$ are defined as

$$\boldsymbol{g}^T=\boldsymbol{G}^T(\boldsymbol{y}_0-\boldsymbol{w}) \qquad (5)$$

$$\boldsymbol{H}=\boldsymbol{G}^T\boldsymbol{G}+\lambda\boldsymbol{I} \qquad (6)$$

Equation (4) gives the whole trajectory of the future control increments and such is an open-loop strategy. To close the loop, only the first element is applied to the system and the whole algorithm is recomputed at time k+1.

If we denote the first row of the matrix $\left(\boldsymbol{G}^T\boldsymbol{G}+\lambda\boldsymbol{I}\right)^{-1}\boldsymbol{G}^T$ as $\boldsymbol{K}$ then the actual control increment can be calculated as

$$\Delta u(k)=\boldsymbol{K}(\boldsymbol{w}-\boldsymbol{y}_0) \tag{7}$$

Predictive control can also handle constraints of input, output and state variables. In this case the optimization problem is a task of quadratic programming.

2.1 Polynomial Methods

Polynomial control theory is based on the apparatus and methods of a linear algebra (see eg. [18] and [19]). The polynomials are the basic tool for a description of the transfer functions. They are expressed as the finite sequence of figures – the coefficients of a polynomial. Thus, the signals are expressed as infinite sequences of figures. The controller synthesis consists in solving of linear polynomial (Diophantine) equations in a general form [24].

The design of the controller algorithm is based on the general block scheme of a closed loop with two degrees of freedom (2DOF) according to Fig. 2.

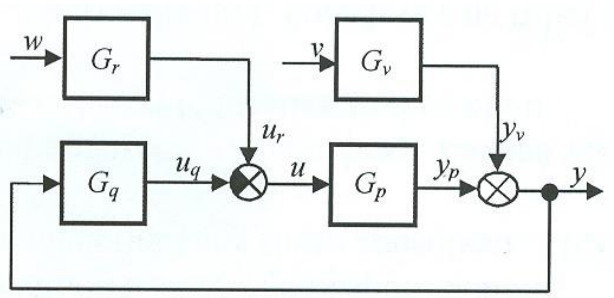

Fig. 2 Block diagram of a closed loop 2DOF control system

The controlled process is given by a transfer function in the form of proper polynomial fractions

$$G_p(z)=\frac{Y_p(z)}{U(z)}=\frac{B(z^{-1})}{A(z^{-1})} \tag{8}$$

Where A and B are coprime polynomials that fulfill the inequality $\deg B \le \deg A$. The controller contains a feedback part G_q and a feedforward part G_r, y is the controlled output, u is the manipulated variable, w is the reference signal and v is the load disturbance with transfer function

$$G_v(z)=\frac{Y_v(z)}{V(z)}=\frac{D(z^{-1})}{A(z^{-1})} \tag{9}$$

The controllers can be also expressed in the form of discrete transfer functions:

$$G_r(z)=\frac{R(z^{-1})}{P(z^{-1})};\quad G_q(z)=\frac{Q(z^{-1})}{P(z^{-1})} \tag{10}$$

The polynomial approach to the design of a control system with the disturbance rejection is used in [25], [26], [27].

The control algorithm is designed for the reference signal tracking and rejection of known sinusoidal disturbance. Step changes of the reference signal are usually used in practice and the sinusoidal disturbance is supposed as it was mentioned in the previous section. Then a step of height w_1 can be expressed as

$$W(z^{-1})=\frac{h_w(z^{-1})}{f_w(z^{-1})}=\frac{w_1}{1-z^{-1}} \tag{11}$$

and the sinusoidal disturbance signal can be expressed as

$$V(z^{-1})=\frac{h_v(z^{-1})}{f_v(z^{-1})}=\frac{A_v\beta z^{-1}}{1-\alpha z^{-1}+z^{-2}} \tag{12}$$

Where A_v is the amplitude of the sinusoidal signal, $\beta=\sin\omega T_0$ and $\alpha=2\cos\omega T_0$; ω and T_0 are the fundamental angular frequency and the fundamental period of the sinusoidal signal.
According to the scheme in Fig. 2 the output can be expressed as:

$$Y(z^{-1})=\frac{G_p(z)G_r(z)}{1+G_p(z)G_q(z)}W(z^{-1})+\frac{G_v(z)}{1+G_p(z)G_q(z)}V(z^{-1}) \tag{13}$$

By combining (8), (9), (10) and (13), expression for the control error can be derived

$$\begin{aligned}&E(z^{-1})=W(z^{-1})-Y(z^{-1})=\\&\frac{A(z^{-1})P(z^{-1})+B(z^{-1})Q(z^{-1})-B(z^{-1})R(z^{-1})}{A(z^{-1})P(z^{-1})+B(z^{-1})Q(z^{-1})}W(z^{-1})-\\&-\frac{D(z^{-1})P(z^{-1})}{A(z^{-1})P(z^{-1})+B(z^{-1})Q(z^{-1})}V(z^{-1})\end{aligned} \tag{14}$$

To ensure the disturbance rejection, the polynomial $P(z^{-1})$ must contain the denominator of $V(z^{-1})$.

$$P(z^{-1})=\tilde{P}(z^{-1})(1-\alpha z^{-1}+z^{-2}) \tag{15}$$

The feedback part of the controller ensures stability of control and disturbance attenuation. It is given by solution of the following Diophantine equation

$$A(z^{-1})f_v(z^{-1})\tilde{P}(z^{-1})+B(z^{-1})Q(z^{-1})=M(z^{-1}) \tag{16}$$

where M is a stable polynomial. The asymptotic tracking is provided by the feedforward part of the controller given by solution of the Diophantine equation

$$T\left(z^{-1}\right)f_w\left(z^{-1}\right)\tilde{P}\left(z^{-1}\right)+B\left(z^{-1}\right)R\left(z^{-1}\right)=M\left(z^{-1}\right) \quad (17)$$

where T is an auxiliary polynomial which does not affect controller design but which is necessary for calculation of (17). The degrees of individual polynomials must fulfill following equalities

$$\begin{aligned} &\deg Q=\deg A+\deg f_v-1\\ &\deg \tilde{P}=\deg A-1\\ &\deg R=\deg f_w-1\\ &\deg T=2\deg A+\deg f_v-\deg f_w-1\\ &\deg D=2\deg A+\deg f_v-1 \end{aligned} \quad (18)$$

The controller parameters then result from solution of polynomial equations (16) and (17) and depend on coefficients of the polynomial M that enables to obtain a suitable stabilizing and stable controller.

3 Model of the System

A model of the second order which is widely applied in practice and has proved to be effective for control of a range of various processes was chosen for the controllers design. It can be expressed by following transfer function

$$G_p(z)=\frac{B\left(z^{-1}\right)}{A\left(z^{-1}\right)}=\frac{b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}} \quad (19)$$

The disturbance is supposed to be modeled by

$$G_v(z)=\frac{D\left(z^{-1}\right)}{A\left(z^{-1}\right)}=\frac{d_1z^{-1}+d_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}} \quad (20)$$

A widely used model in general model predictive control is the CARIMA (controller autoregressive integrated moving average) model which we can obtain by adding a disturbance model as

$$A\left(z^{-1}\right)Y(z)=B\left(z^{-1}\right)U(z)+\frac{C\left(z^{-1}\right)}{\Delta}E_s(z) \quad (21)$$

where E_s is a non-measurable random disturbance that is assumed to have zero mean value and constant covariance and $\Delta=1-z^{-1}$. The colouring polynomial C will be further considered as $\boldsymbol{C}\left(z^{-1}\right)=1$.

Known disturbances can be taken explicitly into account in predictive control. The disturbances are included in prediction equations. In this case model (21) must be changed to include the disturbances

$$A\left(z^{-1}\right)y(k)=B\left(z^{-1}\right)u(k)+D\left(z^{-1}\right)v(k)+\frac{C\left(z^{-1}\right)}{\Delta}e_s(k) \quad (22)$$

where $\boldsymbol{v}(k)$ is a known disturbance and $D(z^{-1})$ is a polynomial defined in (20) as

$$D\left(z^{-1}\right)=d_1z^{-1}+d_2z^{-2} \quad (23)$$

4 Disturbance Rejection in Predictive Control

The known disturbance must be included into computation of systems output predictions. The predictor (2) is then modified to include the measurable disturbances

$$\hat{\boldsymbol{y}}=\boldsymbol{G}\Delta\boldsymbol{u}+\boldsymbol{L}\Delta\boldsymbol{v}+\boldsymbol{y}_0 \quad (24)$$

The forced response is augmented by the term $\boldsymbol{L}\Delta\boldsymbol{v}$ which is that part of the systems response which is determined by future values of increments of the disturbance. The matrix $\boldsymbol{L}$ is defined by the output values of the plant when a step disturbance is introduced. There are several methods how to derive prediction equations. This paper will be focused on two approaches: methods based on Diophantine equations [1] and straightforward computation on the basis of the CARIMA model [28].

Predictive controllers based on both introduced methods for computation of predictions were tested by simulation control of a range of systems. Results obtained for particular methods were compared each other. In all cases were obtained identical results. It means that each method makes the same final prediction equations. Thus in further simulation experiments will not be the particular methods for computation of prediction equations differentiated.

Particular methods will be described in the following subsections.

4.1 Method Based on Diophantine Equations

It is possible to compute j-step ahead prediction from model (22) (for simplification, the operator z^{-1} will be omitted in some expressions)

$$\hat{y}(k+j)=\frac{B}{A}u(k+j)+\frac{D}{A}v(k+j)+\frac{C}{\Delta A}e_s(k+j) \quad (25)$$

From the last term of this expression can be separated terms with positive powers of z where E is a polynomial of the order j minus one and F is a polynomial of the same order as the polynomial A.

$$\frac{C\left(z^{-1}\right)}{\Delta A\left(z^{-1}\right)}=E_j\left(z^{-1}\right)+z^{-j}\frac{F_j\left(z^{-1}\right)}{\Delta A\left(z^{-1}\right)} \quad (26)$$

After substitution to equation (24) we can obtain the predictor in the form

$$\hat{y}(k+j) = \frac{B}{A}u(k+j) + \frac{D}{A}v(k+j) + E_j e_s(k+j) + \frac{F_j}{\Delta A}e_s(k) \quad (27)$$

From original equation (22) we can compute the disturbance and substitute to equation (25)

$$\hat{y}(k+j) = \frac{B}{C}\left[\frac{C}{\Delta A} - z^{-j}\frac{F_j}{\Delta A}\right]\Delta u(k+j) + \frac{D}{C}\left[\frac{C}{\Delta A} - z^{-j}\frac{F_j}{\Delta A}\right]\Delta v(k+j) + \frac{F_j}{C}y(k) + E_j e_s(k+j) \quad (28)$$

After substitution we obtain

$$\hat{y}(k+j) = \frac{BE_j}{C}\Delta u(k+j) + \frac{DE_j}{C}\Delta v(k+j) + \frac{F_j}{C}y(k) + E_j e_s(k+j) \quad (29)$$

Now let us make two simplifications: a white noise case will be considered and future noise values will be further omitted.

$$\hat{y}(k+j) = BE_j\Delta u(k+j) + DE_j\Delta v(k+j) + F_j y(k) \quad (30)$$

We can define polynomials *Gj* ans *Lj* as follows

$$G_j = BE_j \qquad L_j = DE_j \quad (31)$$

$$\hat{y}(k+j) = G_j\Delta u(k+j) + L_j\Delta v(k+j) + F_j y(k) \quad (32)$$

For the design of the j – step ahead predictor the following Diophantine equation is solved

$$1 = E_j\Delta A + z^{-j}F_j \quad (33)$$

Further is necessary to solve a recursion of Diophantine equation (33). Particular polynomials in the Diophantine equation can be expanded as follows

$$\tilde{A}(z^{-1}) = \Delta A(z^{-1}) = 1 + (1-a_1)z^{-1} + (a_1 - a_2)z^{-2} + a_2 z^{-3} \quad (34)$$

$$E_j(z^{-1}) = E_{j,0} + E_{j,1}z^{-1} + E_{j,2}z^{-2} + \cdots + E_{j,j-1}z^{j-1} \quad (35)$$

$$F_j(z^{-1}) = F_{j,0} + F_{j,1}z^{-1} + F_{j,2}z^{-2} + \cdots + F_{j,j-1}z^{j-1} \quad (36)$$

Let us consider the Diophantine equation corresponding to the prediction $\hat{y}(k+j+1)$

$$1 = E_{j+1}(z^{-1})\tilde{A}(z^{-1}) + z^{-(j+1)}F_{j+1}(z^{-1}) \quad (37)$$

It is possible to subtract Diophantine equation (33) from Diophantine equation (37) and obtain the following expression

$$0 = \left(E_{j+1}(z^{-1}) - E_j(z^{-1})\right)\tilde{A}(z^{-1}) + z^{-j}\left(z^{-1}F_{j+1}(z^{-1}) - F_j(z^{-1})\right) \quad (38)$$

Now it is possible to define the following term

$$E_{j+1}(z^{-1}) - E_j(z^{-1}) = \tilde{R}(z^{-1}) + R_j z^{-1} \quad (39)$$

After substitution

$$0 = \tilde{R}(z^{-1})\tilde{A}(z^{-1}) + z^{-j}\left(R_j\tilde{A}(z^{-1}) + z^{-1}F_{j+1}(z^{-1}) - F_j(z^{-1})\right) \quad (40)$$

it is obvious that $\tilde{R}(z^{-1}) = 0$ in order to obtain the zero polynomial on the left side of equation (40). The polynomial *E* can be then computed recursively according to the following expression

$$E_{j+1}(z^{-1}) = E_j(z^{-1}) + R_j z^{-j} \quad (41)$$

Following expressions can be obtained from equation (40)

$$R_j = F_{j,0}$$
$$F_{j+1,i} = F_{j,i+1} - R_j\tilde{A}_{i+1} \quad \text{for} \quad i = 0\cdots\delta(F_{j+1}) \quad (42)$$

Initial conditions for the recursion are as follows

$$E_1 = 1 \qquad F_1 = z(1-\tilde{A}) \quad (43)$$

By making the polynomials

$$E_j(z^{-1})B(z^{-1}) = G_j(z^{-1}) + z^{-j}G_{jp}(z^{-1}) \quad (44)$$

$$E_j(z^{-1})D(z^{-1}) = L_j(z^{-1}) + z^{-j}L_{jp}(z^{-1}) \quad (45)$$

the prediction equation can be written as

$$\hat{y}(k+j) = G_j(z^{-1})\Delta u(k+j) + L_j(z^{-1})\Delta v(k+j) + G_{jp}(z^{-1})\Delta u(k-1) + L_{jp}(z^{-1})\Delta v(k-1) + F_j(z^{-1})y(k) \quad (46)$$

The last three terms of equation (46) depend on past values of the process output, input and disturbance and represent the free response of the process. The first two terms depend on future values of control increments and disturbances and represent the forced response of the system. Equation (46) can be rewritten as

$$\hat{y}(k+j) = G_j(z^{-1})\Delta u(k+j) + L_j(z^{-1})\Delta v(k+j) + y_{0j} \quad (47)$$

Where

$$y_{0j} = G_{jp}(z^{-1})\Delta u(k-1) + L_{jp}(z^{-1})\Delta v(k-1) + F_j(z^{-1})y(k) \quad (48)$$

is the free response.

In case of the second order system, the polynomial $\tilde{A}$ has the following form

$$\tilde{A}(z^{-1}) = \Delta A(z^{-1}) = 1 + (a_1 - 1)z^{-1} + (a_2 - a_1)z^{-2} - a_2 z^{-3} \quad (49)$$

Initial conditions of the recursion are

$$\boldsymbol{E}_1 = 1 \quad (50)$$

$$F_1 = z(1-\tilde{A}) = 1 - a_1 + z^{-1}(a_1 - a_2) + z^{-2}a_2 = f_0 + z^{-1}f_1 + z^{-2}f_2 \quad (51)$$

Initialization of the matrix of the free response and the matrices of the dynamics is following

$$\boldsymbol{x} = \begin{bmatrix} f_0 & f_1 & f_2 & b_2 & d_2 \end{bmatrix} \tag{52}$$

$$\boldsymbol{G} = b_1 \qquad \boldsymbol{L} = d_1 \tag{53}$$

The recursion then proceeds according to previously introduced steps.

$$R = f_0 \tag{54}$$

$$f_0 = f_1 - R(a_1 - 1) \tag{55}$$

$$f_1 = f_2 - R(a_2 - a_1) \tag{56}$$

$$f_2 = Ra_2 \tag{57}$$

$$\boldsymbol{E} = \begin{bmatrix} \boldsymbol{E} & R \end{bmatrix} \tag{58}$$

Extension of the matrices of the dynamics and the free response is as follows:

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{G} \\ b_1\boldsymbol{E}(i+1) + b_2\boldsymbol{E}(i) \end{bmatrix} \tag{59}$$

$$\boldsymbol{L} = \begin{bmatrix} \boldsymbol{G} \\ d_1\boldsymbol{E}(i+1) + d_2\boldsymbol{E}(i) \end{bmatrix} \tag{60}$$

$$\boldsymbol{x} = \begin{bmatrix} & & & \boldsymbol{x} & \\ f_0 & f_1 & f_2 & b_2\boldsymbol{E}(i+1) & d_2\boldsymbol{E}(i+1) \end{bmatrix} \tag{61}$$

4.2 Method Based on Direct Computation from CARIMA Model

This method is based on an analytical derivation of certain predictions and subsequent recursive derivation of later predictions. The number of predictions which are necessary to be computed directly depends on the order of the system. The a priori analytical computation, which is required, enables to reduce computational complexity of the previously introduced method. This is important in the adaptive predictive control where the computation must be performed in each sampling period.

The difference equation of the CARIMA model without the unknown term can be expressed as:

$$\begin{aligned} y(k) = (1-a_1)y(k-1) + (a_1 - a_2)y(k-2) + a_2 y(k-3) + \\ + b_1\Delta u(k-1) + b_2\Delta u(k-2) + d_1\Delta v(k-1) + d_2\Delta v(k-2) \end{aligned} \tag{62}$$

It was necessary to directly compute three step ahead predictions in a straightforward way by establishing of previous predictions to later predictions. The model order defines that computation of one step ahead prediction is based on the three past values of the system output. The prediction equation (24) after modification can be written in a matrix form

$$\begin{aligned} \overbrace{\begin{bmatrix} \hat{y}(k+1) \\ \hat{y}(k+2) \\ \hat{y}(k+3) \end{bmatrix}}^{\hat{\boldsymbol{y}}} &= \overbrace{\begin{bmatrix} g_1 & 0 \\ g_2 & g_1 \\ g_3 & g_2 \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \end{bmatrix}}^{\boldsymbol{G}\Delta\boldsymbol{u}} + \\ &+ \overbrace{\begin{bmatrix} l_1 & 0 \\ l_2 & l_1 \\ l_3 & l_2 \end{bmatrix} \begin{bmatrix} \Delta v(k) \\ \Delta v(k+1) \end{bmatrix}}^{\boldsymbol{L}\Delta\boldsymbol{v}} + \\ &+ \overbrace{\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} \end{bmatrix} \begin{bmatrix} y(k) \\ y(k-1) \\ y(k-2) \\ \Delta u(k-1) \\ \Delta v(k-1) \end{bmatrix}}^{\boldsymbol{y}_0} \end{aligned} \tag{63}$$

It is possible to divide computation of the predictions to recursion of the free response and recursion of the matrices of the dynamics.

All the elements $p(i,j)$ i=1…3, j=1…5 have to be directly computed to initialize the recursion.

$$\begin{aligned} & p_{11} = (1-a_1) \quad p_{12} = (a_1 - a_2) \quad p_{13} = a_2 \quad p_{14} = b_2 \quad p_{15} = d_2 \\ & p_{21} = (1-a_1)^2 + (a_1 - a_2) \quad p_{22} = (1-a_1)(a_1 - a_2) + a_2 \\ & p_{23} = a_2(1-a_1) \quad p_{24} = (1-a_1)b_2 \quad p_{25} = (1-a_1)d_2 \\ & p_{31} = (1-a_1)^3 + 2(1-a_1)(a_1-a_2) + a_2 \\ & p_{32} = (1-a_1)^2(a_1-a_2) + a_2(1-a_1) + (a_1-a_2)^2 \\ & p_{33} = a_2(1-a_1)^2 + (a_1-a_2)a_2 \\ & p_{34} = (1-a_1)^2 b_2 + (a_1-a_2)b_2 \\ & p_{35} = (1-a_1)^2 d_2 + (a_1-a_2)d_2 \end{aligned} \tag{64}$$

The next row of the free response matrix is repeatedly computed on the basis of the three previous predictions until the prediction horizon is achieved.

$$\begin{aligned} p_{41} &= (1-a_1)p_{31} + (a_1-a_2)p_{21} + a_2 p_{11} \\ p_{42} &= (1-a_1)p_{32} + (a_1-a_2)p_{22} + a_2 p_{12} \\ p_{43} &= (1-a_1)p_{33} + (a_1-a_2)p_{23} + a_2 p_{13} \\ p_{44} &= (1-a_1)p_{34} + (a_1-a_2)p_{24} + a_2 p_{14} \\ p_{45} &= (1-a_1)p_{35} + (a_1-a_2)p_{25} + a_2 p_{15} \end{aligned} \tag{65}$$

The forced response in equation (63) has following form:

$$\boldsymbol{G}\Delta\boldsymbol{u} = \begin{bmatrix} b_1 & 0 \\ b_1(1-a_1)+b_2 & b_1 \\ (a_1-a_2)b_1+(1-a_1)^2 b_1+(1-a_1)b_2 & (1-a_1)b_1+b_2 \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \end{bmatrix} \quad (66)$$

$$\boldsymbol{L}\Delta\boldsymbol{v} = \begin{bmatrix} d_1 & 0 \\ d_1(1-a_1)+d_2 & d_1 \\ (a_1-a_2)b_1+(1-a_1)^2 d_1+(1-a_1)d_2 & (1-a_1)d_1+d_2 \end{bmatrix} \begin{bmatrix} \Delta v(k) \\ \Delta v(k+1) \end{bmatrix} \quad (67)$$

The recursion of the matrices $\boldsymbol{G}$ and $\boldsymbol{L}$ is similar to the recursion of the free response matrix. The next element of the first column is repeatedly computed and the remaining columns are shifted. This procedure is performed repeatedly until the prediction horizon is achieved. If the control horizon is lower than the prediction horizon a number of columns in the matrix $\boldsymbol{G}$ is reduced. Computation of new elements is performed as follows:

$$\begin{aligned} g_4 &= (1-a_1)g_3 + (a_1-a_2)g_2 + a_2 g_1 \\ l_4 &= (1-a_1)l_3 + (a_1-a_2)l_2 + a_2 l_1 \end{aligned} \quad (68)$$

5 Disturbance Rejection by Polynomial Methods

Degrees of the particular polynomials in the control loop are obtained from equations (18).

$$\begin{aligned} &\deg Q = \deg A + \deg f_v - 1 = 2+2-1 = 3 \\ &\deg \tilde{P} = \deg A - 1 = 2-1 = 1 \\ &\deg R = \deg f_w - 1 = 1-1 = 0 \\ &\deg T = 2\deg A + \deg f_v - \deg f_w - 1 = 4+2-1-1 = 4 \\ &\deg D = 2\deg A + \deg f_v - 1 = 4+2-1 = 5 \end{aligned} \quad (69)$$

Consequently, the particular polynomials are in the following form

$$\begin{aligned} &Q(z^{-1}) = q_0 + q_1 z^{-1} + q_2 z^{-2} + q_3 z^{-3} \\ &\tilde{P}(z^{-1}) = 1 + p_1 z^{-1} \\ &R(z^{-1}) = r_0 \\ &T(z^{-1}) = t_0 + t_1 z^{-1} + t_2 z^{-2} + t_3 z^{-3} + t_4 z^{-4} \\ &D(z^{-1}) = m_0 + m_1 z^{-1} + m_2 z^{-2} + m_3 z^{-3} + m_4 z^{-4} + m_5 z^{-5} \end{aligned} \quad (70)$$

After substitution of polynomials (70) to Diophantine equation (16) we can obtain a system of linear equations with unknown controllers parameters

$$\begin{bmatrix} b_1 & 0 & 0 & 0 & 1 \\ b_2 & b_1 & 0 & 0 & a_1-\alpha \\ 0 & b_2 & b_1 & 0 & 1-\alpha a_1+a_2 \\ 0 & 0 & b_2 & b_1 & a_1-\alpha a_2 \\ 0 & 0 & 0 & b_2 & a_2 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \\ p_1 \end{bmatrix} = \begin{bmatrix} m_1+\alpha-a_1 \\ m_2-a_2-1+\alpha a_1 \\ m_3-a_1+\alpha a_2 \\ m_4-a_2 \\ m_5 \end{bmatrix} \quad (71)$$

Similar approach can be used for Diophantine equation (17) to obtain the parameter r_0

$$r_0 = \frac{1+m_1+m_2+m_3+m_4+m_5}{b_1+b_2} \quad (72)$$

The control law which ensues from Fig. 2 and transfer functions (10) is then given as

$$\begin{aligned} u(k) &= r_0 w(k) - q_0 y(k) - q_1 y(k-1) - q_2 y(k-2) - q_3 y(k-3) - \\ &- (p_1-\alpha)u(k-1) - (1-\alpha p_1)u(k-2) - p_1 u(k-3) \end{aligned} \quad (73)$$

6 Simulation Examples

Both controllers were tested by simulation control of a range of systems. Control of the following system is given as an example

$$G_p(z) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{0{,}20z^{-1}+0{,}17z^{-2}}{1-1{,}51z^{-1}+0{,}55z^{-2}} \quad (74)$$

$$G_v(z) = \frac{D(z^{-1})}{A(z^{-1})} = \frac{0{,}10z^{-1}-0{,}20z^{-2}}{1-1{,}51z^{-1}+0{,}55z^{-2}} \quad (75)$$

A sinusoid of angular frequency 1 rad/sec and amplitude 1 was applied as the disturbance. Tuning parameters of the predictive controller are the weighting factor λ and the prediction and control horizons. The controller based on polynomial methods has as tuning parameters poles of the polynomial D.

The performances of both controllers were compared by means of control quality criteria, which are the sum of powers of control errors and the sum of increments of manipulated variables.

For both controllers it is possible to set the rate of changes in the manipulated variable (for the predictive controller by the parameter λ and for the controller based on polynomial methods by poles of the polynomial D). As larger changes in manipulated variable as better quality of asymptotic tracking of reference signal is achieved. However, large changes of manipulated variable are often undesirable. In order to compare the performances of both controllers, the rate of changes of the manipulated variable was set to be approximately the same in both cases. For the polynomial controller a suitable multiple pole 0,2 was found. The predictive controller was tuned by the weighting factor to achieve approximately the same sum of increments of the manipulated variable. It was achieved for $\lambda = 0{,}077$.

In Fig. 3, Fig. 4, Fig. 7 and Fig. 8 are time responses of control without the disturbance. In Fig. 9 and Fig. 10 are time responses of control with the predictive controller with the disturbance when the prediction equations do not include information about the disturbance. The controller based on polynomial methods was designed for the specific shape of the disturbance and thus it is not possible to simulate control without the disturbance rejection. In Fig. 5, Fig. 6, Fig. 11, Fig. 12, Fig. 13 and Fig. 14 are time responses with the disturbance when both controllers take into account the disturbance. It is obvious that the influence of disturbance was suppressed.

In case of predictive controller, the objective function (1) was used for computation of control sequence. We considered an unconstrained case even though possibility of incorporation of constraints is very important in predictive control since one of the main advantages of predictive control is its ability to deal effectively with constraints. But the paper is focused on another part of predictive control: computation of predictions with incorporation of known disturbance. So that the simulated control problem was simplified to be unconstrained. In this case computation of optimal control is a direct problem of linear algebra.

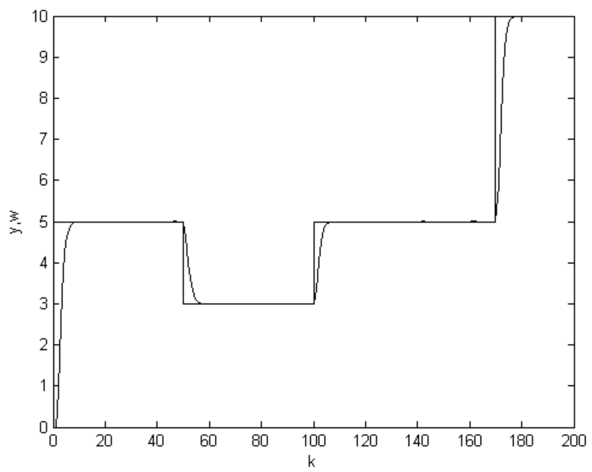

Fig. 3 Controller based on polynomial methods-control without disturbance

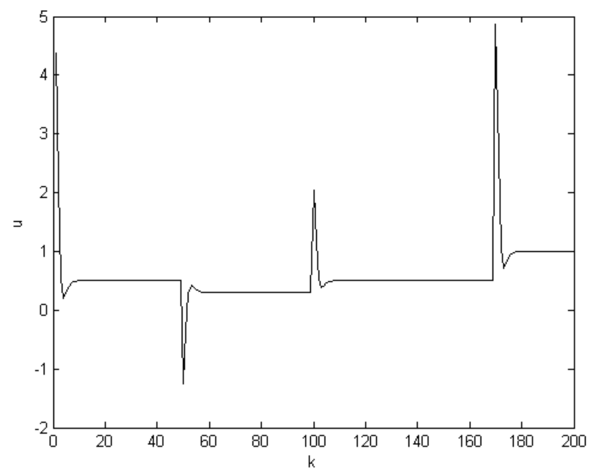

Fig. 4 Controller based on polynomial methods-control without disturbance-manipulated variable

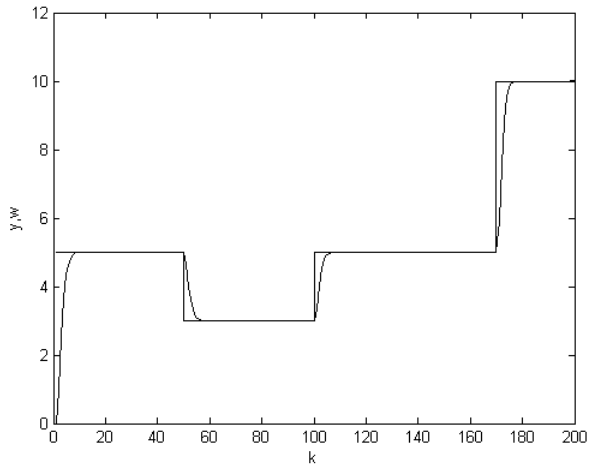

Fig. 5 Controller based on polynomial methods-control with disturbance with disturbance rejection

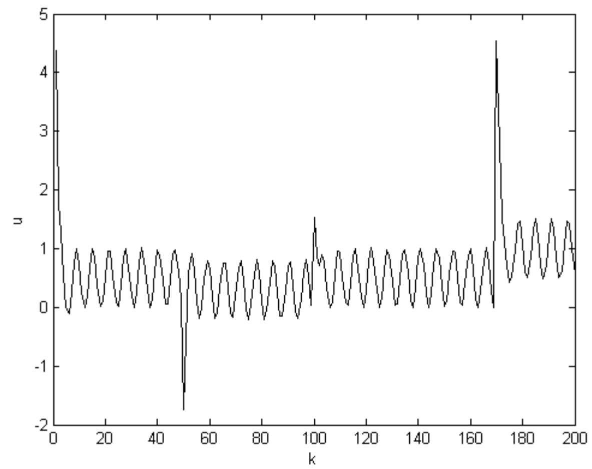

Fig. 6 Controller based on polynomial methods-control with disturbance with disturbance rejection-manipulated variable

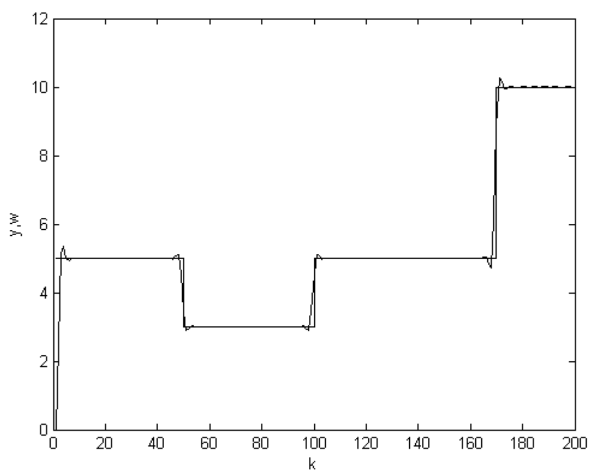

Fig. 7 Predictive control without disturbance $\lambda = 0{,}01$

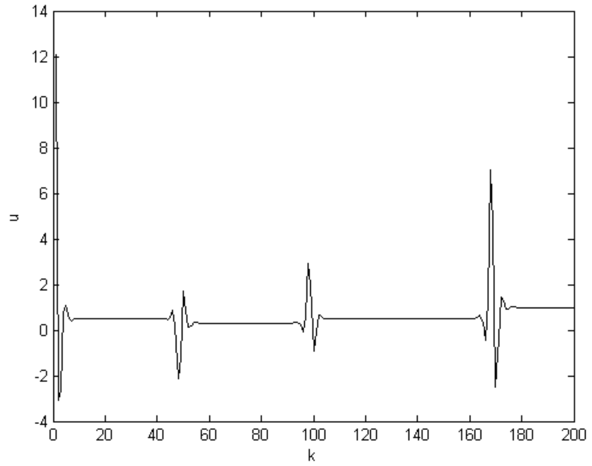

Fig. 8 Predictive control without disturbance $\lambda = 0{,}01$-manipulated variable

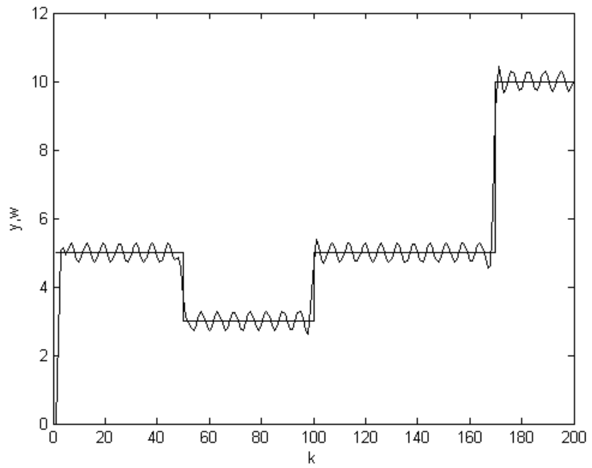

Fig. 9 Predictive controller $\lambda = 0{,}01$- control with disturbance without disturbance rejection

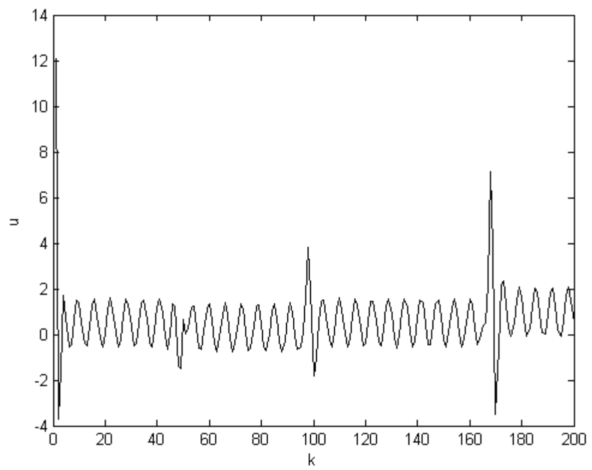

Fig. 10 Predictive controller $\lambda = 0{,}01$- control with disturbance without disturbance rejection-manipulated variable

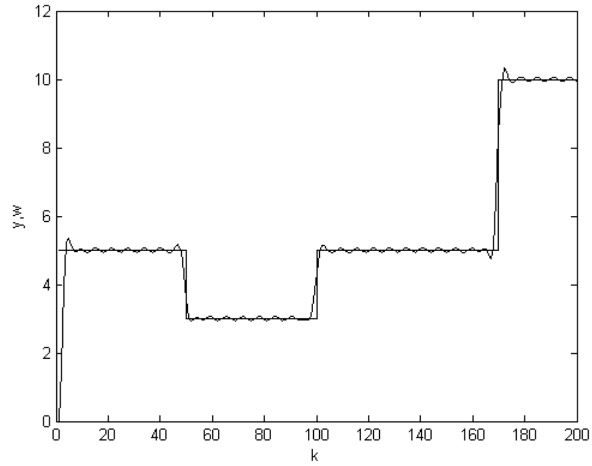

Fig. 11 Predictive controller $\lambda = 0{,}077$ - control with disturbance with disturbance rejection

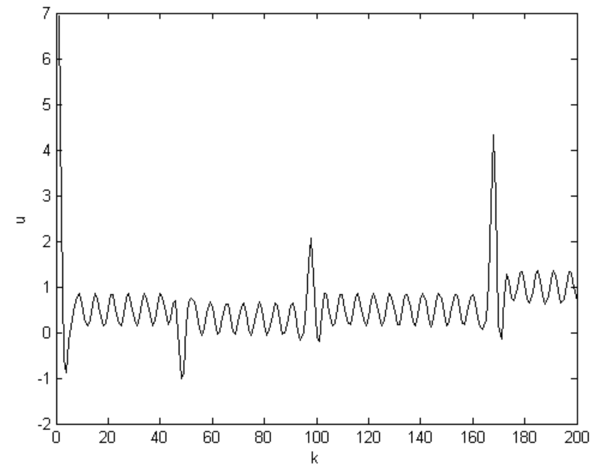

Fig. 12 Predictive controller $\lambda = 0{,}077$ - control with disturbance with disturbance rejection-manipulated variable

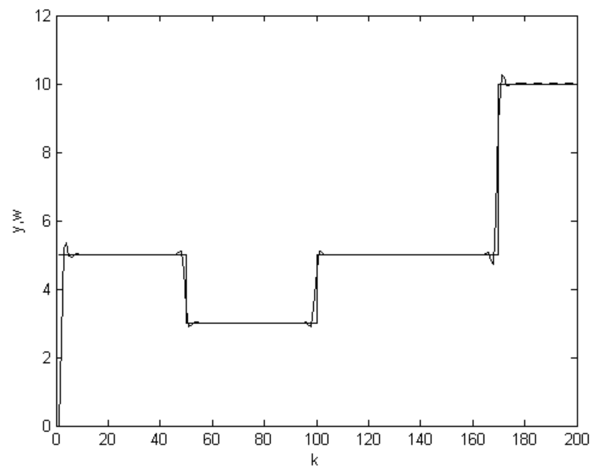

Fig. 13 Predictive controller $\lambda = 0{,}01$- control with disturbance with disturbance rejection

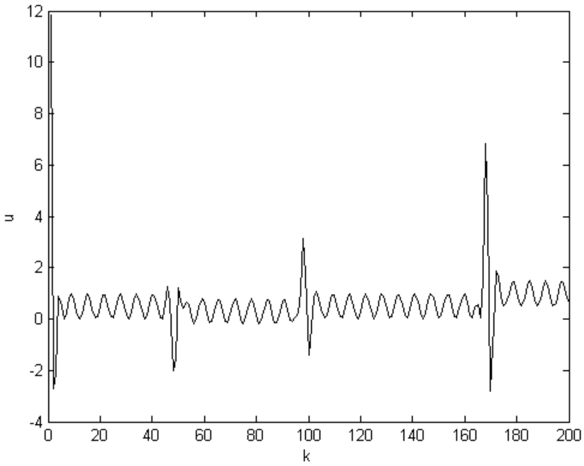

Fig. 14 Predictive controller $\lambda = 0{,}01$ - control with disturbance with disturbance rejection-manipulated

Table. 1 Control Quality Criteria

Controller	$\sum e^2$	$\sum \Delta u^2$
Polynomial	115,04	64,79
Predictive $\lambda = 0{,}077$	45,32	64,79
Predictive $\lambda = 0{,}01$	37,12	271,03

7 Conclusions

Two different control algorithms which enable suppression of measurable disturbances were proposed and compared. If a controller based on polynomial methods is applied then for each shape of the disturbance a different controller must be derived. For the predictive controller it is possible to put into a general prediction equation an arbitrary disturbance. The polynomial controller for sinusoidal disturbance was derived and performances of both controllers were compared by simulation. The simulation results proved that both controllers can be successfully applied for disturbance suppression. According to the chosen control quality criteria better performance has the predictive controller. On the other hand the controlled variable has slightly oscillatory character when using the predictive controller. The oscillations can be suppressed by larger rate of changes of the manipulated variable, which is however often undesirable.

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