Control Design of a Nonlinear Multivariable Process

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Abstract: – The paper presents an effective procedure for control design of multi input – multi output nonlinear processes. The procedure is based on an approximation of a nonlinear model of the process by a continuous-time external linear model in the form of the left polynomial matrix fraction. The parameters of the continuous-time external linear model are recursively estimated either by a direct method or through an external delta model. The control system structure with two feedback controllers is used. The controllers are derived using the explicit pole assignment method. The control is simulated on the nonlinear model of two conic liquid tanks in series.

Key-Words: – Nonlinear system, multivariable system, polynomial matrix, external linear model, delta model, polynomial approach, pole assignment.

1 Introduction

A most part of processes in chemical, biochemical, polymer and other technologies exhibits nonlinear properties. From the system theory, these processes belong to the class of nonlinear systems. Moreover, a certain part of such processes requires to control more output signals independently. In order to achieve this, it is necessary to have at least as many independent input signals as output signals to be controlled. Such processes are classified as multivariable or multi-input multi-output (MIMO) processes.

It is well known that the control of nonlinear MIMO processes often represents very complex problem and traditional methods based on a use of controllers with fixed parameters can lead to control of a poor quality. In this case, it is necessary to apply some of the so called advanced methods. Here, the procedures can be based on internal state space or external input-output descriptions. As a frequently used method may be mentioned Model predictive control, e.g. [1] and [2], Nonlinear control, e.g. [3], LQ control, e.g. [4], [5], Robust control, e.g. [6]. The other methods can be found e.g. in [7], [8], [9], [10], and [11]..

One possible method to cope with this problem is using adaptive strategies based on an appropriate choice of an external linear model (ELM) in the left polynomail matrix fraction description with recursively estimated parameters. which are consequently used for parallel updating of the controller‘s parameters.

Two basic approaches can be used for identification of the continuous-time (CT) ELM. The first direct method [12], [13] and [14] is based on filtration of input and output signals where the filtered variables have the same properties (in the *s*-domain) as their non-filtered counterparts. Derivatives of filtered signals that are necessary for the parameters estimate of the CT ELM are obtained from differential filters. This method has, however, some drawbacks – the necessity to solve additional differential equations representing the filters and estimate time constants of these filters. The second strategy uses an external δ-model of the controlled process with the same structure as a CT model. The basics of δ-models have been described e.g. in [15] and [16]. Here, parameters of δ-models can directly be estimated from sampled signals without the necessity to filter them. Moreover, it can be easily proved that these parameters converge to parameters of CT models for a sufficiently small sampling period (compared to the dynamics of the controlled process), see e.g. [17]. The control results obtained using both mentioted strategies were compared for the single-input single-output (SISO) system in [18]. This paper presents full control design procedure of a nonlinear MIMO process. The parameters of the CT ELM of the process are identified by both above mentioted methods. The control structure with two feedback controllers is used according to [19] and [20]. Input signals for the control system are step references and step load disturbances. Resulting controllers are derived using the polynomial approach [21], [22] and the pole placement method, e.g. [23] and [24] with operations carried out in the ring of polynomial matrices.

Note that in the presence of a time delay, the methods described e.g. in [25] – [29] may be used

2 CT External Linear Model

In the time domain, the generalized continuous-time ELM is specified by the vector differential equation

$$\boldsymbol{A}(\sigma)\boldsymbol{y}(t) = \boldsymbol{B}(\sigma)\boldsymbol{u}(t) \tag{1}$$

where $\sigma = d/dt$ is the derivative operator, $\boldsymbol{y} \in \Re^r$ stands for the controlled output vector, $\boldsymbol{u} \in \Re^m$ is the control input vector and $\boldsymbol{A}$, $\boldsymbol{B}$ are polynomial matrices in σ. Using the Laplace transform, the model is described in the *s*-domain as

$$\boldsymbol{A}(s)\boldsymbol{Y}(s) = \boldsymbol{B}(s)\boldsymbol{U}(s) + \boldsymbol{o}_1(s) \tag{2}$$

where $\boldsymbol{o}_1$(s) is the vector of initial conditions, and, $\boldsymbol{A}(s) \in \Re^{rr}[s]$ and $\boldsymbol{B}(s) \in \Re^{rm}[s]$ are left coprime polynomial matrices in the form

$$\boldsymbol{A}(s) = \begin{pmatrix} a_{11}(s) & .. & a_{1i}(s) & .. & a_{1r}(s) \\ : & & : & & : \\ a_{i1}(s) & .. & a_{ii}(s) & .. & a_{ir}(s) \\ : & & : & & : \\ a_{r1}(s) & .. & a_{ri}(s) & .. & a_{rr}(s) \end{pmatrix} \tag{3}$$

$$\boldsymbol{B}(s) = \begin{pmatrix} b_{11}(s) & .. & b_{1m}(s) & .. & b_{1m}(s) \\ : & & : & & : \\ b_{i1}(s) & .. & b_{ii}(s) & .. & b_{im}(s) \\ : & & : & & : \\ b_{r1}(s) & .. & b_{ri}(s) & .. & b_{rm}(s) \end{pmatrix}. \tag{4}$$

The polynomials in matrices $\boldsymbol{A}(s)$ and $\boldsymbol{B}(s)$ are in general forms

$$a_{ij}(s) = a_{n_{ij},ij} s^{n_{ij}} + a_{n_{ij}-1,ij} s^{n_{ij}-1} + ... + a_{1,ij} s + a_{0,ij} \tag{5}$$

$$b_{ik}(s) = b_{m_{ik},ik} s^{m_{ik}} + ... + b_{1,ik} s + b_{0,ik} \tag{6}$$

for $i, j = 1, \dots, r$ and $k = 1, \dots, m$.

The transfer function of the controlled system is assumed in the form of the left polynomial matrix fraction

$$\boldsymbol{G}(s) = \boldsymbol{A}^{-1}(s)\boldsymbol{B}(s). \tag{7}$$

Further, consider strictly proper $\boldsymbol{G}(s)$, and, with regard to some following operations, assume that the highest power of *s* in each row of the matrix $\boldsymbol{A}$ lies on its diagonal. Moreover, the monic polynomials with the unit coefficient by the highest power of *s* on the diagonal are assumed ($a_{n_{ii},ii} = 1$).

Remark: The degree of the *i*-th row of a polynomial matrix $\boldsymbol{M}$ is $r_i\, \boldsymbol{M} = \max_j n_{ij}$. Then, for the matrix $\boldsymbol{A}$ with a highest power of *s* on diagonal the relations $r_i\, \boldsymbol{A} = n_{ii}$ and $\deg A = n = \max_i n_{ii}$ hold.

3 CT ELM Parameter Estimation

The direct method of the CT ELM parameter estimation can be briefly carried out as follows.

Since the derivatives of all input and output cannot be directly measured, vectors of filtered variables $\boldsymbol{u}_f$ and $\boldsymbol{y}_f$ are established as the outputs of filters

$$\boldsymbol{C}(\sigma)\boldsymbol{u}_f(t) = \boldsymbol{u}(t) \tag{8}$$

$$\boldsymbol{C}(\sigma)\boldsymbol{y}_f(t) = \boldsymbol{y}(t) \tag{9}$$

where $\boldsymbol{C}$(σ) is a stable polynomial matrix in σ that fulfills the condition

$$\deg \boldsymbol{C}(\sigma) \geq \deg \boldsymbol{A}(s) = n \tag{10}$$

where the sign of equality is mostly used.

Now, using the *L*-transform of (8) and (9), the expressions

$$\boldsymbol{C}(s)\boldsymbol{U}_f(s) = \boldsymbol{U}(s) + \boldsymbol{o}_2(s) \tag{11}$$

$$\boldsymbol{C}(s)\boldsymbol{Y}_f(s) = \boldsymbol{Y}(s) + \boldsymbol{o}_3(s) \tag{12}$$

can be obtained where $\boldsymbol{o}_2$ and $\boldsymbol{o}_3$ are polynomial vectors of initial conditions. Substituting (11) and (12) into (2), the relation for filtered output takes the form

$$\boldsymbol{A}(s)\boldsymbol{C}(s)\boldsymbol{Y}_f(s) = \boldsymbol{B}(s)\boldsymbol{C}(s)\boldsymbol{U}_f(s) + \boldsymbol{o}(s) \tag{13}$$

where

$$\boldsymbol{o}(s) = \boldsymbol{o}_1(s) - \boldsymbol{B}(s)\boldsymbol{o}_2(s) + \boldsymbol{A}(s)\boldsymbol{o}_3(s). \tag{14}$$

The next procedure requires the matrix $\boldsymbol{C}$(s) in the diagonal form

$$\boldsymbol{C}(s) = c(s)\boldsymbol{I}_r \tag{15}$$

where $\boldsymbol{I}_r$ is the unit matrix and *c*(*s*) is a monic polynomial of degree *n*.

Then, the relation between filtered variables has the form

$$\boldsymbol{A}(s)\boldsymbol{Y}_f(s) = \boldsymbol{B}(s)\boldsymbol{U}_f(s) + \boldsymbol{\psi}(s) \tag{16}$$

where $\boldsymbol{\psi}(s) = \boldsymbol{o}(s)/c(s)$.

A comparison of (2) and (16) shows equality of transfer behaviour of filtered and nonfiltered variables.

Remark: $\boldsymbol{\psi}(s)$ is a vector of rational functions as the transforms of a vector function $\boldsymbol{\psi}(t)$ which expresses a difference between initial conditions of filtered and nonfiltered variables (in reference to a last steady state).

After conversion of (16) to the time domain, the equation for filtered variables takes the form

$$\boldsymbol{A}(\sigma)\,\boldsymbol{y}_f(t) = \boldsymbol{B}(\sigma)\boldsymbol{u}_f(t)\,. \tag{17}$$

The equation describing i-th row of (17) can be written as

$$\begin{aligned}&\sum_{j=0}^{n_{i1}} a_{j,i1}\, y_{1f}^{(j)} + \ldots + \sum_{j=0}^{n_{ii}} a_{j,ii}\, y_{if}^{(j)} + \ldots + \sum_{j=0}^{n_{ir}} a_{j,ir}\, y_{rf}^{(j)} = \\ &= \sum_{j=0}^{m_{i1}} b_{j,i1} u_{1f}^{(j)} + \ldots + \sum_{j=0}^{m_{im}} b_{j,im} u_{mf}^{(j)}\end{aligned}\,. \tag{18}$$

Now, the filtered variables including their derivatives can be sampled from filters (8) and (9) in discrete time intervals $t_k = k\,T_S$, $k = 0,1,2,\ldots$ where T_S is the sampling period.

Introducing the regression vector

$$\begin{aligned}\boldsymbol{\Phi}_i^T(t_k) = \Big(&-y_{1f}(t_k) \ldots - y_{1f}^{(n_{i1})}(t_k), \ldots, \\ &-y_{if}(t_k) \ldots - y_{if}^{(n_{ii}-1)}(t_k), \ldots, -y_{rf}(t_k) \ldots - y_{rf}^{(n_{ir})}(t_k), \\ &u_{1f}(t_k), \ldots u_{1f}^{(m_{i1})}(t_k), \ldots, u_{mf}(t_k) \ldots u_{mf}^{(m_{im})}(t_k)\Big)\end{aligned} \tag{19}$$

the vector of parameters in the i-th row

$$\begin{aligned}\boldsymbol{\Theta}_i^T = \Big(&a_{0,i1} \ldots a_{n_{i1},i1}, \ldots, a_{0,ii} \ldots a_{n_{ii}-1,ii}, \ldots, \\ &a_{0,ir} \ldots a_{n_{ir},ir}, b_{0,i1} \ldots b_{m_{i1},i1}, \ldots, b_{0,im} \ldots b_{m_{im},im}\Big)\end{aligned} \tag{20}$$

can then be estimated in discrete times from the ARX model, see, e.g. [30] and [31].

$$y_{if}^{(n_{ii})}(t_k) = \boldsymbol{\Theta}_i(t_k)\,\boldsymbol{\Phi}_i(t_k) + \varepsilon_i(t_k)\,. \tag{21}$$

4 Delta External Linear Model

Establish the δ-operator defined by

$$\delta = \frac{q-1}{T_0} \tag{22}$$

where q is the forward shift operator and T_0 is the sampling interval. When the sampling interval is shortened, the δ-operator approaches the derivative operator σ so that

$$\lim_{T_0 \to 0} \delta = \sigma \tag{23}$$

and, the δ-model

$$\boldsymbol{A}'(\delta)\,\boldsymbol{y}(t') = \boldsymbol{B}'(\delta)\boldsymbol{u}(t') \tag{24}$$

approaches the continuous-time model (1). Here, t' is the discrete time, and, $\boldsymbol{A}'$ and $\boldsymbol{B}'$ are matrices with an identical structure as $\boldsymbol{A}$ and $\boldsymbol{B}$ in the form

$$\boldsymbol{A}'(\delta) = \begin{pmatrix} a'_{11}(\delta) & .. & a'_{1i}(\delta) & .. & a'_{1r}(\delta) \\ : & & : & & : \\ a'_{i1}(\delta) & .. & a'_{ii}(\delta) & .. & a'_{ir}(\delta) \\ : & & : & & : \\ a'_{r1}(\delta) & .. & a'_{ri}(\delta) & .. & a'_{rr}(\delta) \end{pmatrix} \tag{25}$$

$$\boldsymbol{B}'(\delta) = \begin{pmatrix} b'_{11}(\delta) & .. & b'_{1m}(\delta) & .. & b'_{1m}(\delta) \\ : & & : & & : \\ b'_{i1}(\delta) & .. & b'_{ii}(\delta) & .. & b'_{im}(\delta) \\ : & & : & & : \\ b'_{r1}(\delta) & .. & b'_{ri}(\delta) & .. & b'_{rm}(\delta) \end{pmatrix} \tag{26}$$

with polynomials

$$\begin{aligned}a'_{ij}(\delta) = a'_{n_{ij},ij}\,\delta^{n_{ij}} + a'_{n_{ij}-1,ij}\,\delta^{n_{ij}-1} + \ldots \\ \ldots + a'_{1,ij}\,\delta + a'_{0,ij}\end{aligned} \tag{27}$$

$$b'_{ik}(\delta) = b'_{m_{ik},ik}\,\delta^{m_{ik}} + \ldots + b'_{1,ik}\delta + b'_{0,ik} \tag{28}$$

where

$a'_{n_{ii},ii} = 1$, $n_{ii} > n_{ij}$ for $j \neq i$ and $n_{ii} > m_{ik}$ for all i, $j = 1, \ldots, r$ and $k = 1, \ldots, m$.

Substituting $t' = k_0 - n_{ii}$ where $k_0 \geq n_{ii}$, the equation describing i-th row of (24) can be derived as

$$\begin{aligned}&\sum_{j=0}^{n_{i1}} a'_{j,i1}\delta^j y_1(k_0 - n_{ii}) + \ldots \\ &+ \sum_{j=0}^{n_{ii}} a'_{j,ii}\delta^j y_i(k_0 - n_{ii}) + \ldots \\ &+ \sum_{j=0}^{n_{ir}} a'_{j,ir}\delta^j y_r(k_0 - n_{ii}) = \\ &= \sum_{j=0}^{m_{i1}} b'_{j,i1}\delta^j u_1(k_0 - n_{ii}) + \ldots \\ &+ \sum_{j=0}^{m_{im}} b'_{j,im}\delta^j u_m(k_0 - n_{ii})\end{aligned} \tag{29}$$

where the terms in (29) are

$$\delta^{n_{ij}} y_i(k_0 - n_{ii}) = = \sum_{p=0}^{n_{ij}} \frac{(-1)^p}{T_0^{n_{ij}}} \binom{n_{ij}}{p} y_i(k_0 - n_{ii} + n_{ij} - p) \quad (30)$$

$$\delta^{m_{ik}} u_k(k_0 - n_{ii}) = = \sum_{p=0}^{m_{ik}} \frac{(-1)^p}{T_0^{m_{ik}}} \binom{m_{ik}}{p} u_k(k_0 - n_{ii} + m_{ik} - p). \quad (31)$$

5 Delta ELM parameter estimation

Obviously, an actual value of the controlled output $y_i(k_0)$ in the i-th row is only in the term $\delta^{n_{ii}} y_i(k_0 - n_{ii})$ (for $j = i$ and $p = 0$ in (30)). Now, denoting

$$\varphi_{i,y_i}^j = \delta^j y_i(k_0 - n_{ii}), \ \varphi_{i,u_k}^j = \delta^j u_k(k_0 - n_{ii}) \quad (32)$$

and, introducing the regression vector

$$\boldsymbol{\Phi}_{\delta i}^T = \Big(-\varphi_{i,y_1}^0 \dots -\varphi_{i,y_1}^{n_{i1}}, \dots, -\varphi_{i,y_i}^0 \dots -\varphi_{i,y_i}^{n_{ii}-1}, \dots, -\varphi_{i,y_r}^0 \dots -\varphi_{i,y_r}^{n_{ir}}, \varphi_{i,u_1}^0 \dots \varphi_{i,u_1}^{m_{i1}}, \dots, \varphi_{i,u_m}^0 \dots \varphi_{i,u_m}^{m_{im}} \Big) \quad (33)$$

then, the vector of parameters in the i-th row of A'

$$\boldsymbol{\Theta}_i^T = \Big(a'_{0,i1} \dots a'_{n_{i1},i1}, \dots, a'_{0,ii} \dots a'_{n_{ii}-1,ii}, \dots, a'_{0,ir} \dots a'_{n_{ir},ir}, b'_{0,i1} \dots b'_{m_{i1},i1}, \dots, b'_{0,im} \dots b'_{m_{im},im} \Big) \quad (34)$$

can be recursively estimated from the regression (ARX) model

$$\varphi_{i,y_i}^{n_{ii}} = \boldsymbol{\Theta}_i^T \boldsymbol{\Phi}_{\delta i} + \varepsilon_i(k_0). \quad (35)$$

or, in detail, from the equation

$$\delta^{n_{ii}} y_i(k_0 - n_{ii}) = -\sum_{j=0}^{n_{i1}} a'_{j,i1} \delta^j y_1(k_0 - n_{ii}) - \dots - \sum_{j=0}^{n_{ii}-1} a'_{j,ii} \delta^j y_i(k_0 - n_{ii}) - \dots \sum_{j=0}^{n_{ir}} a'_{j,ir} \delta^j y_r(k_0 - n_{ii}) + + \sum_{j=0}^{m_{i1}} b'_{j,i1} \delta^j u_1(k_0 - n_{ii}) + \dots + \sum_{j=0}^{m_{im}} b'_{j,im} \delta^j u_m(k_0 - n_{ii}) + \varepsilon_i(k_0). \quad (36)$$

6 Controller Design

The control system with two feedback controllers is depicted in Fig. 1. Here, $\boldsymbol{G}$ represents the CT ELM, $\boldsymbol{G}_Q$ and $\boldsymbol{G}_R$ are controllers.

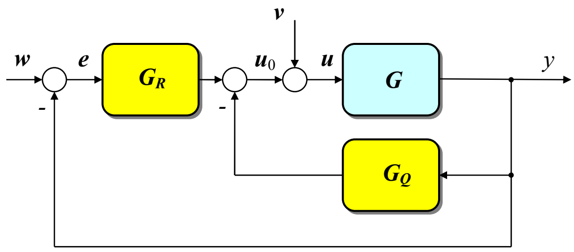

Fig. 1: Control system structure.

Further, $\boldsymbol{w} \in \Re^r$ is the vector of references and $\boldsymbol{v} \in \Re^m$ is the vector of load disturbances. Generally, their transforms can be expressed as

$$\boldsymbol{w}(s) = \boldsymbol{F}_w^{-1}(s)\boldsymbol{h}_w(s), \ \boldsymbol{v}(s) = \boldsymbol{F}_v^{-1}(s)\boldsymbol{h}_v(s). \quad (37)$$

Considering all elements of both input signals as step function, matrices $\boldsymbol{F}_w$ and $\boldsymbol{F}_v$ in (37) take forms

$$\boldsymbol{F}_w(s) = \boldsymbol{F}_v(s) = s\,\boldsymbol{I} \quad (38)$$

and vectors (37) can be rewritten to

$$\boldsymbol{W}(s) = \left(\frac{w_{10}}{s} \ \frac{w_{20}}{s} \ \dots \ \frac{w_{r0}}{s} \right)^T \quad (39)$$

$$\boldsymbol{V}(s) = \left(\frac{v_{10}}{s} \ \frac{v_{20}}{s} \ \dots \ \frac{v_{m0}}{s} \right)^T \quad (40)$$

where w_{i0} and v_{j0} are constants.

The transfer functions of controllers are assumed in the form of right coprime polynomial matrix fractions

$$\boldsymbol{G}_Q(s) = \boldsymbol{Q}_1(s)\boldsymbol{P}_1^{-1}(s), \quad \boldsymbol{G}_R(s) = \boldsymbol{R}_1(s)\boldsymbol{P}_1^{-1}(s) \quad (41)$$

where $\boldsymbol{Q}_1(s) \in \Re^{mr}[s]$, $\boldsymbol{R}_1(s) \in \Re^{mr}[s]$ and $\boldsymbol{P}_1(s) \in \Re^{rr}[s]$.

The goal is to find such proper controllers that ensure the control system stability, asymptotic tracking of step references and step load disturbance attenuation. The procedure for deriving admissible controllers can be performed as follows: Using descriptions of basic signals in the control system

$$\boldsymbol{y}(s) = \boldsymbol{A}^{-1}\boldsymbol{B}\,\boldsymbol{u}(s) = \boldsymbol{A}^{-1}\boldsymbol{B}\left[\boldsymbol{u}_0(s) + \boldsymbol{v}(s)\right] \quad (42)$$

$$\boldsymbol{u}_0(s) = \boldsymbol{R}_1 \boldsymbol{P}_1^{-1} \left[\boldsymbol{w}(s) - \boldsymbol{y}(s) \right] - \boldsymbol{Q}_1 \boldsymbol{P}_1^{-1} \boldsymbol{y}(s) \quad (43)$$

the output and tracking error vectors can be derived as

$$\boldsymbol{y}(s) = \boldsymbol{P}_1 \boldsymbol{D}^{-1} \left[\boldsymbol{B} \boldsymbol{R}_1 \boldsymbol{P}_1^{-1} \boldsymbol{w}(s) + \boldsymbol{B} \boldsymbol{v}(s) \right] \quad (44)$$

$$\boldsymbol{e}(s) = \boldsymbol{P}_1 \boldsymbol{D}^{-1} \left[(\boldsymbol{A}\boldsymbol{P}_1 + \boldsymbol{B}\boldsymbol{Q}_1) \boldsymbol{P}_1^{-1} \boldsymbol{w}(s) - \boldsymbol{B} \boldsymbol{v}(s) \right] \quad (45)$$

vhere

$$\boldsymbol{D} = \boldsymbol{A}\boldsymbol{P}_1 + \boldsymbol{B}(\boldsymbol{R}_1 + \boldsymbol{Q}_1). \quad (46)$$

Now, feedback controllers given by a solution of the matrix Diophantine equation

$$\boldsymbol{A}\boldsymbol{P}_1 + \boldsymbol{B}\boldsymbol{T} = \boldsymbol{D} \quad (47)$$

with a stable polynomial matrix $\boldsymbol{D} \in \Re^{rr}[s]$ on the right side that ensures the control system stability. Here, the matrix $\boldsymbol{T}$ has been established as

$$\boldsymbol{T} = \boldsymbol{R}_1 + \boldsymbol{Q}_1. \quad (48)$$

The step load disturbances will be rejected for the matrix $\boldsymbol{P}_1$ in (45) divisible by denominators s in (39) and (40). This condition is fulfilled for $\boldsymbol{P}_1$ in the form

$$\boldsymbol{P}_1(s) = s\,\tilde{\boldsymbol{P}}_1(s). \quad (49)$$

Asymptotic tracking of step references is ensured for the term $\boldsymbol{A}\boldsymbol{P}_1 + \boldsymbol{B}\boldsymbol{Q}_1$ divisible by s in denominators of (39). Evidently, this divisibility is fulfilled for $\boldsymbol{Q}_1$ taking the form

$$\boldsymbol{Q}_1(s) = s\,\tilde{\boldsymbol{Q}}_1(s). \quad (50)$$

Taking into account (49) and (50), polynomial matrices of controllers are given by a solution of the matrix Diophantine equation

$$\boldsymbol{A}(s)\, s\,\tilde{\boldsymbol{P}}_1(s) + \boldsymbol{B}(s)\boldsymbol{T}(s) = \boldsymbol{D}(s) \quad (51)$$

where

$$\boldsymbol{T}(s) = \boldsymbol{R}_1(s) + s\,\tilde{\boldsymbol{Q}}_1(s). \quad (52)$$

Evidently, the degrees of matrices are given as

$$\deg \boldsymbol{R}_1 = \deg \boldsymbol{T}, \ \deg \tilde{\boldsymbol{Q}}_1 = \deg \boldsymbol{T} - 1. \quad (53)$$

Considering expansions of matrices $\boldsymbol{T}$, $\boldsymbol{R}_1$ and $\tilde{\boldsymbol{Q}}_1$ as

$$\boldsymbol{T}(s) = \sum_{j=0}^{\deg \boldsymbol{T}} s^j \boldsymbol{T}_j \quad (54)$$

$$\boldsymbol{R}_1(s) = \sum_{j=0}^{\deg \boldsymbol{T}} s^j \boldsymbol{R}_{1j} \quad (55)$$

$$\tilde{\boldsymbol{Q}}_1(s) = \sum_{j=1}^{\deg \boldsymbol{T}} s^{j-1} \tilde{\boldsymbol{Q}}_{1j} \quad (56)$$

where $\boldsymbol{T}_j$, $\boldsymbol{R}_{1j}$ and $\tilde{\boldsymbol{Q}}_{1j}$ are matrices of constant coefficients, a solution of (51) leads to a simple term of $\boldsymbol{T}$ given by

$$\boldsymbol{B}_0 \boldsymbol{T}_0 = \boldsymbol{D}_0 \quad (57)$$

and, subsequently, to

$$\boldsymbol{R}_{10} = \boldsymbol{T}_0. \quad (58)$$

It is well known that a solution of a single polynomial matrix equation provides only two unknown polynomial matrices. Hence, selectable coefficient matrices $\boldsymbol{\beta}_j \in \Re^{mm}$ can be introduced that distribute weights among $\boldsymbol{R}_1$ and $\tilde{\boldsymbol{Q}}_1$ parameters. Denoting expansions of matrices $\boldsymbol{R}_1$ and $\tilde{\boldsymbol{Q}}_1$ as

$$\boldsymbol{R}_{1j}, \ \tilde{\boldsymbol{Q}}_{1j}, \quad j = 1, \ldots, \deg \boldsymbol{T} \quad (59)$$

then, their elements can be calculated from equations

$$\boldsymbol{R}_{1j} = \boldsymbol{\beta}_j \boldsymbol{T}_j, \ \tilde{\boldsymbol{Q}}_{1j} = \left(\boldsymbol{I} - \boldsymbol{\beta}_j\right)\boldsymbol{T}_j \quad (60)$$

for $j = 1, \ldots, \deg \boldsymbol{T}$.

Remark: If $\boldsymbol{\beta}_j = \boldsymbol{I}$ for all j, the control system in Fig. 1 simplifies to the 1DOF control configuration. If $\boldsymbol{\beta}_j = \boldsymbol{0}$ for all j, and, both references and load disturbances are step functions, the control system corresponds to the 2DOF control configuration.

From the practical point of view, it is effective to choose $\boldsymbol{\beta}_j$ as diagonal matrices

$$\boldsymbol{\beta}_j = \begin{pmatrix} \beta_{j1} & .. & .. & 0 \\ : & \beta_{j2} & & \\ : & & .. & \\ 0 & & & \beta_{jm} \end{pmatrix} \quad (61)$$

for all j.

Now, taking into account (49) and (50), transfer functions of controllers can be rewritten to the form

$$\boldsymbol{G}_Q(s) = \tilde{\boldsymbol{Q}}_1(s)\left(\tilde{\boldsymbol{P}}_1(s)\right)^{-1} \quad (62)$$

$$G_R(s)=R_1(s)\left(s\tilde{P}_1(s)\right)^{-1}. \qquad (63)$$

Note that degrees of polynomial matrices in transfer functions of controllers must be determined in accordance with the requirement on properness of controller transfer functions.

7 Example and Simulation Results

Two conic liquid tanks in series are considered according to Fig. 2.

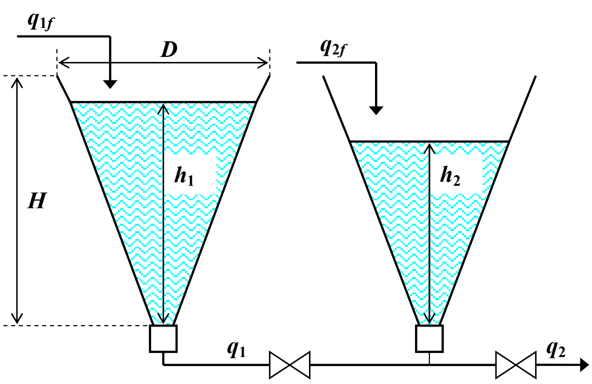

Fig.2 Two conic liquid tanks in series.

Using standard simplifications, the model of the plant can be described by two nonlinear differential equations

$$\pi\frac{D^2}{4H^2}h_1^2\frac{dh_1}{dt}+q_1=q_{1f} \qquad (64)$$

$$\pi\frac{D^2}{4H^2}h_2^2\frac{dh_2}{dt}-q_1+q_2=q_{2f} \qquad (65)$$

where D is the upper diameter of both tanks, H denotes the total high of both tanks, h_j are liquid levels in tanks, q_j stand for stream flowrates and q_{jf} are their inlet values, (for j = 1, 2). The stream volumetric flowrates depend upon levels in tanks as

$$q_1=k_1\sqrt{|h_1-h_2|},\quad q_2=k_2\sqrt{h_2} \qquad (66)$$

$$(\text{if } h_1-h_2<0 \text{ then } q_1=-q_1)$$

where k_1, k_2 are constants.

Initial conditions for (64) and (65) are steady state liquid levels $h_1(0)=h_1^s$, $h_2(0)=h_2^s$. The model parameters and values of variables at the operating point used in simulations are: $k_1=0.316\,\mathrm{m}^{2.5}/\mathrm{min}$, $k_2=0.296\,\mathrm{m}^{2.5}/\mathrm{min}$, D = 1.5 m H = 2.5 m, $h_1^s=1.8\,\mathrm{m}$, $h_2^s=1.4\,\mathrm{m}$, $q_{1f}^s=0.2\,\mathrm{m}^3/\mathrm{min}$, and $q_{2f}^s=0.15\,\mathrm{m}^3/\mathrm{min}$. Both the control and controlled variables are considered to be deviations from their values at the operating point

$$u_1(t)=q_{1f}(t)-q_{1f}^s,\quad u_2(t)=q_{2f}(t)-q_{2f}^s \qquad (67)$$

$$y_1(t)=h_1(t)-h_1^s,\quad y_2(t)=h_2(t)-h_2^s. \qquad (68)$$

Simulated step responses of the process are shown in Figs. 3 and 4.

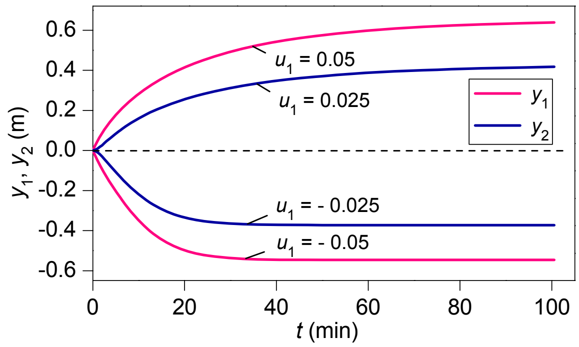

Fig. 3. Controlled outputs step responses to u_1.

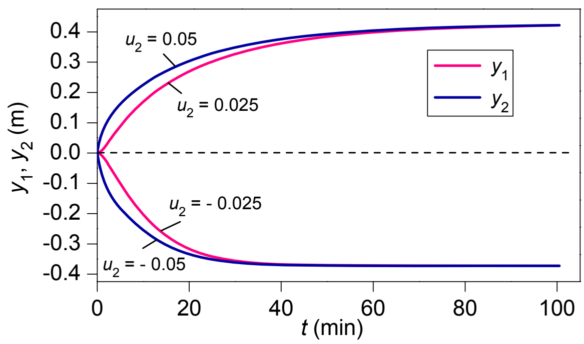

Fig. 4. Controlled outputs step responses to u_2.

Taking into account profiles of these responses, polynomial matrices of the CT external linear model in the form of LPMF have been chosen as

$$A(s)=\begin{pmatrix} s+a_{01} & a_{02} \\ a_{03} & s+a_{04}\end{pmatrix},\quad B(s)=\begin{pmatrix} b_{01} & 0 \\ 0 & b_{04}\end{pmatrix} \qquad (69)$$

Remark: For simplicity, the indexing 1 – 4 has been here used.

In the first case, the parameters in (69) were estimated by the direct method. There, the filtered variables were computed as outputs from the first order filters

$$\dot{y}_{if}+c_0y_{if}=y_i,\quad \dot{u}_{if}+c_0u_{if}=u_i,\quad i=1,2. \qquad (70)$$

Then, the CT ELM parameters were in parallel estimated from two regresion equations

$$\begin{aligned}\dot{y}_{1f}(t_k) &= b_{01}u_{1f}(t_k) - a_{01}y_{1f}(t_k) - \\ &- a_{03}y_{2f}(t_k) + \varepsilon_1(t_k) \\ \dot{y}_{2f}(t_k) &= b_{04}u_{2f}(t_k) - a_{02}y_{1f}(t_k) - \\ &- a_{04}y_{2f}(t_k) + \varepsilon_2(t_k)\end{aligned} \tag{71}$$

in discrete time intervals $t_k = k\,T_s$, $k = 0,1,\ldots$.

In the second case, parameters in () were estimated using a δ-model with corresponding matrices

$$\boldsymbol{A}'(\delta) = \begin{pmatrix} \delta + a'_{01} & a'_{02} \\ a'_{03} & \delta + a'_{04} \end{pmatrix}, \boldsymbol{B}'(\delta) = \begin{pmatrix} b'_{01} & 0 \\ 0 & b'_{04} \end{pmatrix} \tag{72}$$

There, two parallel identifications in the form

$$\delta y_1(k_0-1) = b'_{01}u_1(k_0-1) - a'_{01}y_1(k_0-1) - a'_{03}y_2(k_0-1) + \varepsilon_1(k_0) \tag{73}$$

$$\delta y_2(k_0-1) = b'_{04}u_1(k_0-1) - a'_{02}y_1(k_0-1) - a'_{04}y_2(k_0-1) + \varepsilon_2(k_0) \tag{74}$$

were used where

$$\delta y_i(k_0-1) = \frac{y_i(k_0) - y_i(k_0-1)}{T_0}, \; i = 1,2 \; . \tag{75}$$

for $k_0 = 0,1,\ldots$.

In both cases, the recursive identification method with exponential and directional forgetting according to [20] was used.

With regard to requirement of the controller properness, matrices $\boldsymbol{P}_1$ and $\boldsymbol{T}$ were chosen in the form

$$\boldsymbol{P}_1(s) = s\,\tilde{\boldsymbol{P}}_1(s) = \begin{pmatrix} s\,p_{01} & s\,p_{02} \\ s\,p_{03} & s\,p_{04} \end{pmatrix} \tag{76}$$

$$\boldsymbol{T}(s) = \begin{pmatrix} t_{11}s + t_{01} & t_{12}s + t_{02} \\ t_{13}s + t_{03} & t_{14}s + t_{04} \end{pmatrix} \tag{77}$$

and, the diagonal matrix on the right side of (51) as

$$\boldsymbol{D}(s) = \begin{pmatrix} (s+\alpha_1)^2 & 0 \\ 0 & (s+\alpha_2)^2 \end{pmatrix}. \tag{78}$$

Then, solving (51), the coefficients in (76) and (77) were derived as

$$p_{02} = p_{03} = 0\,, \;\; p_{01} = p_{04} = 1$$

$$t_{01} = \frac{\alpha_1^2}{b'_{01}}, \;\; t_{11} = \frac{1}{b'_{01}}(2\alpha_1 - a'_{01})$$

$$t_{02} = t_{03} = 0\,, \; t_{12} = -\frac{a'_{02}}{b'_{01}}\,, \; t_{13} = -\frac{a'_{03}}{b'_{04}} \tag{79}$$

$$t_{04} = \frac{\alpha_2^2}{b'_{04}}, \;\; t_{14} = \frac{1}{b'_{04}}(2\alpha_2 - a'_{04})\,.$$

Choosing the matrix (61) as

$$\boldsymbol{\beta}_1 = \begin{pmatrix} \beta_{11} & 0 \\ 0 & \beta_{12} \end{pmatrix} \tag{80}$$

and, solving (60), transfer functions of controllers take forms

$$\boldsymbol{G}_Q(s) = \begin{pmatrix} (1-\beta_{11})t_{11} & (1-\beta_{11})t_{12} \\ (1-\beta_{12})t_{13} & (1-\beta_{12})t_{14} \end{pmatrix} \tag{81}$$

$$\boldsymbol{G}_R(s) = \begin{pmatrix} \beta_{11}t_{11} + \dfrac{t_{01}}{s} & \beta_{11}t_{12} \\ \beta_{12}t_{13} & \beta_{12}t_{14} + \dfrac{t_{04}}{s} \end{pmatrix} \tag{82}$$

All simulation experiments were performed for references $w_1 = 0.2, w_2 = 0.15$ in the time interval $0 \le t < 100\,\text{min}$, $w_1 = -0.1, w_2 = 0$ in the time interval $100 \le t < 200\,\text{min}$ and $w_1 = 0.15, w_2 = 0.10$ in the time interval $200 \le t \le 300\,\text{min}$.

For the direct CT ELM parameter estimation, the filter parameter was chosen as c_0 = 0.5 and the sampling period as $T_s = 0.5\,\text{min}$.

The recursive estimation of the delta ELM parameters was performed with the sampling interval T_0 = 0.5 min. For the start, P-controllers with a small gain were used.

The simulation results obtained using the direct CT ELM parameter estimation (designated as CT ID) are in Figs. 5 – 9.

An effect of parameters α on controlled outputs and control inputs is shown in Figs. 5 – 6. Their higher values accelerate the control but lead to overshoots (undershoots) of the controlled outputs. Moreover, higher values of α result in greater changes of control inputs. This fact can be important in control of real processes.

Each output can be influenced differently by selecting different values of α as shown in Fig. 7.

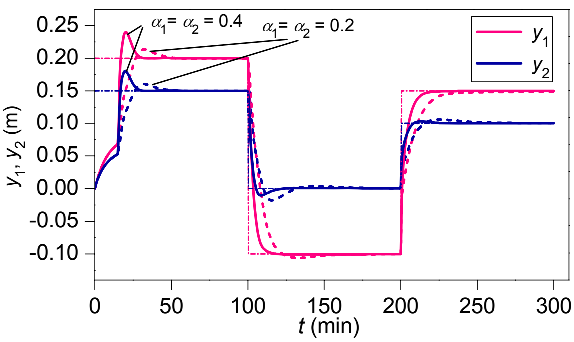

Fig. 5. CT ID: Controlled outputs for various α ($\beta_{11} = \beta_{12} = 0.5$).

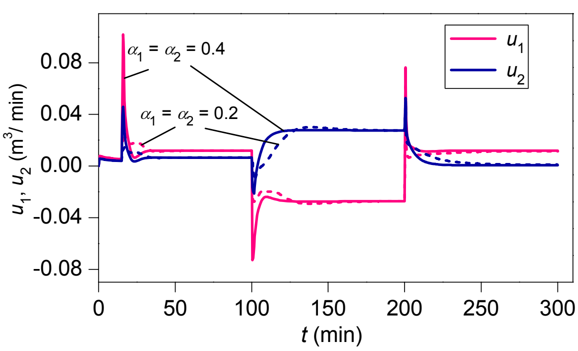

Fig. 6. CT ID: Control inputs for various α ($\beta_{11} = \beta_{12} = 0.5$).

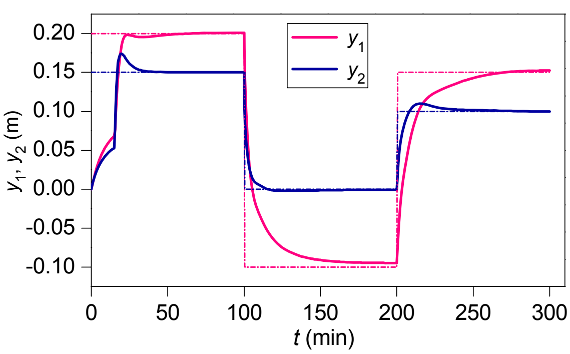

Fig. 7. CT ID: Controlled outputs for $\alpha_1 = 0.2$, $\alpha_2 = 0.4$, $\beta_{11} = \beta_{12} = 0.5$.

An effect of parameters β on controlled outputs and control inputs is shown in Figs. 8 – 9. Here, extreme values of β were considered so that they correspond to the 1DOF ($\beta_{11} = \beta_{12} = 1$) and to the 2DOF control system structure ($\beta_{11} = \beta_{12} = 0$). The results confirm the known fact that the 2DOF structure provides smooth control responses without significant overshoot and leads to more careful control inputs.

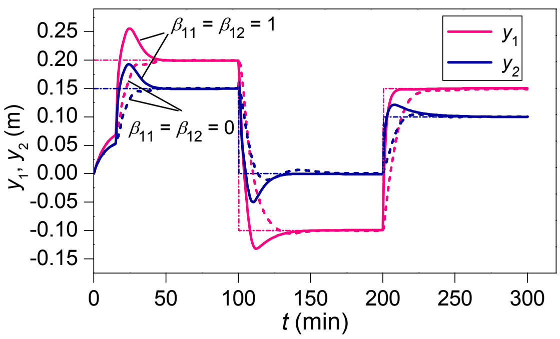

Fig. 8. CT ID: Controlled outputs for various β ($\alpha_1 = \alpha_2 = 0.25$).

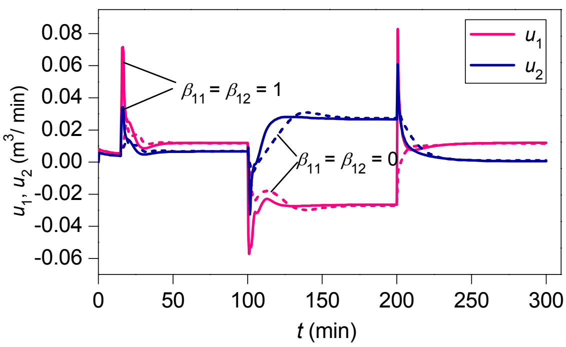

Fig. 9. CT ID: Control inputs for various β ($\alpha_1 = \alpha_2 = 0.25$).

The simulation results obtained using the delta ELM parameter estimation (designated as Delta ID) are presented in Figs. 10 – 13.

The simulation performed with the same parameters α and β as in Fig. 5 is shown in Fig. 10. There are minimal differences between control responses obtained by both identification methods. Both controlled outputs and control inputs can also be shaped by selection of different values of β_{11} and β_{12} as shown in Figs. 11 and 12.

The responses in Fig. 13 show that an appropriate selection of parameters α and β enables to achieve the control of very good quality.

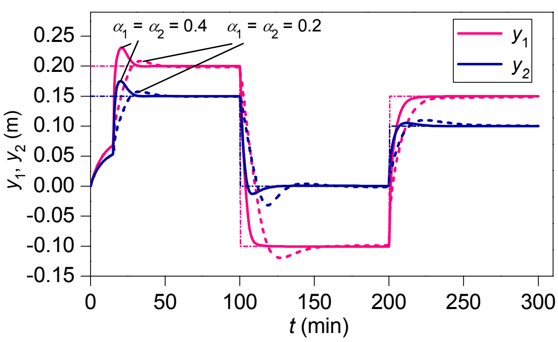

Fig. 10. Delta ID: Controlled outputs for various α ($\beta_{11} = \beta_{12} = 0.5$).

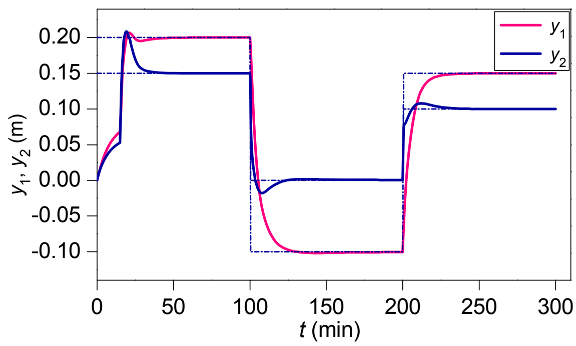

Fig. 11. Delta ID: Controlled outputs for $\alpha_1 = \alpha_2 = 0.4$, $\beta_{11} = 0.1$, $\beta_{12} = 0.9$.

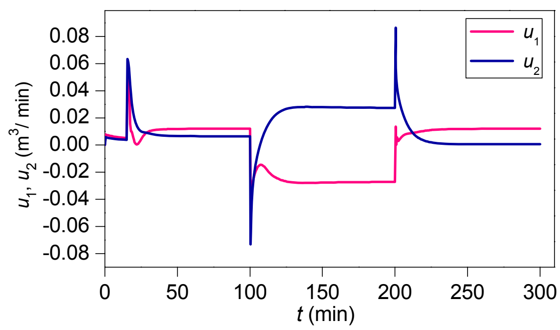

Fig. 12. Delta ID: Control inputs for $\alpha_1 = \alpha_2 = 0.4$, $\beta_{11} = 0.1$, $\beta_{12} = 0.9$.

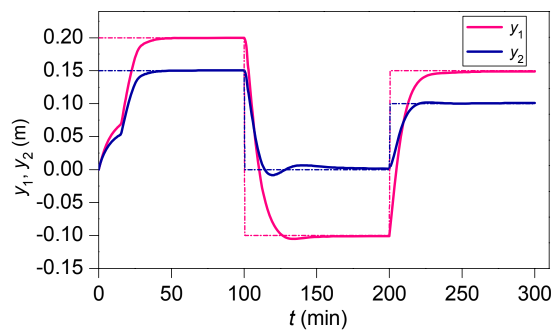

Fig. 13. Delta ID: Controlled outputs for $\alpha_1 = \alpha_2 = 0.25$, $\beta_{11} = \beta_{12} = 0$.

8 Conclusion

The paper presents one approach to the continuous-time adaptive control of nonlinear multi-input multi-output processes. The control design is based on approximation of a nonlinear model of the process by a continuous-time external linear model in the form of the left polynomial matrix fraction. Its parameters are recursively estimated either by a direct method or through an external delta model with a corresponding structure. The control system structure with two feedback controllers is used. Both resulting continuous-time controllers are solved and derived in the ring of polynomial matrices. Parameters of the controllers are periodically readjusted according to recursively estimated parameters of the external linear model. The control quality is ensured by selectable poles of the closed-loop as well as by parameters distributing weights among numerators of the subcontroller transfer functions. The presented method has been tested by computer simulation on the nonlinear model of two conic liquid tanks in series.

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