Annals of DAAAM for 2009 & Proceedings of the 20th International DAAAM Symposium, Volume 20, No. 1, ISSN 1726-9679
ISBN 978-3-901509-70-4, Editor B. Katalinic, Published by DAAAM International, Vienna, Austria, EU, 2009
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Annals of DAAAM International

SIMPLE APPROACH TO CONTROL OF MULTIVARIABLE CONTROL LOOPS

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Abstract: *This paper describes one of possible method to control of MIMO control loop with utilisation of the so called binding members and correction members. The designed method combinations classical way to ensure of autonomy of control loop via binding members and the use of the method of SISO branched control loops with measurement of dominant disturbance variables to ensure of invariance of control loop via correction members. Simulation verification of the method was carried out for three-variable loop of a steam turbine.*
Key words: *autonomy, invariance, MIMO control loop, synthesis*

1. INTRODUCTION

At large numbers of controlled systems (steam boilers, turbines, air-conditioning plants, reactors, etc.) several variables have to be controlled at the same time. In this case there is not larger member of independent SISO (single-variable) control loop. These control loops are complex with several controlled variables where separate variables are not mutually independent. Mutual coupling of controlled variables is usually given by simultaneous action of each of input (manipulated and disturbance) variables of controlled plant to all controlled variables. These control loops are called MIMO (multi-variable) control loops and they are a complex of mutually influencing simpler control loops. (Balda et al., 1968).

2. MIMO CONTROL LOOP

We will consider MIMO control loop with measurement of disturbance according to Fig. 1.

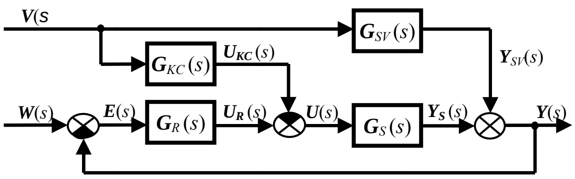

Fig. 1. MIMO control loop with measurement of disturbance

Legend: $\boldsymbol{G}_S(s)$, $\boldsymbol{G}_{SV}(s)$, $\boldsymbol{G}_R(s)$ and $\boldsymbol{G}_{KC}(s)$ - transfer matrix of a controlled plant, disturbance variables, controller and correction members; $\boldsymbol{Y}(s)$ $[n\times1]$ - vector of controlled variables, $\boldsymbol{U}(s)$ $[n\times1]$ - vector of manipulated variables, $\boldsymbol{V}(s)$ $[m\times1]$ - vector of disturbance variables; $m \leq n$

Transfer matrixes of controlled plant and disturbance variables are considered in forms

$$\boldsymbol{G}_S(s)=\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \qquad \boldsymbol{G}_{SV}(s)=\begin{bmatrix} S_{V11} & S_{V12} & \cdots & S_{V1m} \\ S_{V21} & S_{V22} & \cdots & S_{V2m} \\ \vdots & \vdots & \cdots & \vdots \\ S_{Vn1} & S_{Vn2} & \cdots & S_{Vnm} \end{bmatrix} \tag{1}$$

Transfer matrixes of controller and correction member are considered in forms

$$\boldsymbol{G}_R(s)=\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix} \qquad \boldsymbol{G}_{KC}(s)=\begin{bmatrix} KC_{11} & 0 & \cdots & 0 \\ 0 & KC_{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & KC_{nn} \end{bmatrix} \tag{2}$$

2.1 Autonomy and invariance

At synthesis of MIMO control loop, beside stability and quality of control, it is often required for control loop to be autonomous and invariant. In order to determine the conditions for autonomy and invariance we start from a command transfer matrix $\boldsymbol{G}_{W/Y}(s)$ and disturbance transfer matrix $\boldsymbol{G}_{V/Y}(s)$ (Balate, 2004).

$$\boldsymbol{G}_{W/Y}(s) = \left[\boldsymbol{I}+\boldsymbol{G}_S(s)\boldsymbol{G}_R(s)\right]^{-1}\boldsymbol{G}_S(s)\boldsymbol{G}_R(s) \tag{3}$$

$$\boldsymbol{G}_{V/Y}(\mathrm{s})=\left[\boldsymbol{I}+\boldsymbol{G}_S(s)\boldsymbol{G}_R(s)\right]^{-1}\left[\boldsymbol{G}_{SV}(s)-\boldsymbol{G}_S(s)\boldsymbol{G}_{KC}(s)\right] \tag{4}$$

Autonomy of control loop

For ensuring autonomy it is necessary that the matrix $\boldsymbol{G_S}(s)\,\boldsymbol{G_R}(s)$ (3) is diagonal. On the base of this condition it is possible to determined the following relation

$$\frac{R_{kl}}{R_{ml}}=\frac{s_{lk}}{s_{lm}} \qquad k,l,m=<1,\ldots,n>,\ s_{lm}\neq 0 \tag{5}$$

s_{lk}, s_{lm} - algebraic supplements of separate elements of a transfer matrix of controlled plant $\boldsymbol{G}_S(s)$

R_{kl}, R_{ml} - separate members of a transfer matrix of controller $\boldsymbol{G}_R(s)$

It is consider that diagonal (main) controllers $R_{11}, R_{22},\ldots, R_{nn}$ are usually known already from the first design of conception of control. The relation (5) is used for calculation of all remaining members of matrix controller $\boldsymbol{G_R}(s)$, i.e. for calculation of transfers of binding members.

Invariance of control loop

For ensuring invariance it is necessary that the disturbance transfer matrix $G_{V/Y}(s)$ (4) is zero. This is possible if the following relation is valid

$$\boldsymbol{G}_{KC}(s)=\boldsymbol{G}_S^{-1}(s)\boldsymbol{G}_{SV}(s) \tag{6}$$

At design of correction members KC, the task of which is to eliminate the influence of disturbance variable on control loop, internal couplings are omitted at MIMO control loop and thus n SISO branched control loop with measurement of a disturbance variable are gained. Connection of all these SISO control loops is the same and they differ only in separate transfers of controlled plants, controllers, correction members and disturbance variables (Balate, 2004) (see Fig. 2).

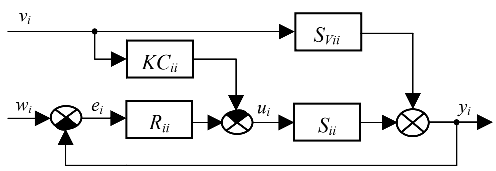

Fig. 2. Block diagram of SISO branched control loop with measurement of disturbance variable v_i

The invariance of the control loop is ensured, according to the above mentioned method, by using analogy of SISO branched control loops with measurement of disturbance variable v. Transfer of correction members KC are gained by using the equation (6) in the following form

$$KC_{ii}=\frac{S_{Vii}}{S_{ii}} \qquad i=<1,\ldots,n>,\ S_{ii}\neq 0 \tag{7}$$

S_{Vii} - separate members of transfer matrix of disturbance variables $\boldsymbol{G_{SV}}(s)$

S_{ii} - separate members of transfer matrix of controlled plant $\boldsymbol{G_S}(s)$

2.2 Synthesis of MIMO control loop

In practice the possible approximate solution of MIMO control loop is applied from analysis of MIMO control loop and really used control schemes in particular technological equipments (Balate, 2004). One of the possible methods of solution of MIMO control loops synthesis is described in the following part:

- Design of **main controllers (diagonal controllers)** by any synthesis method of SISO control loops, i.e. design of parameters of main controllers for n SISO control loops ($R_{11}, R_{22}, \ldots, R_{nn}$). Here, it is considered that original diagonal transfer functions S_{ii} $(i=1,\ldots,n)$ of transfer matrix of controlled plant $\boldsymbol{G}_S(s)$ are modified to diagonal transfer functions $S_{ii,x}$ $(i=1,\ldots,n)$. In these modified transfer functions influences of aside-from diagonal transfer functions of transfer matrix of controlled plant $\boldsymbol{G}_S(s)$, i.e. S_{ij} $(i\neq j,\ i,j=1,\ldots,n)$ on original diagonal transfer functions i.e. S_{ii} $(i=1,\ldots,n)$ are included. Transfer functions $S_{ii,x}$, i.e $S_{11,x}$, $S_{22,x}$, $S_{33,x}$ etc. are determined from equation (8) by using relations (3) and (5).

$$S_{ii,x}=\sum_{j=1}^{n} S_{ij}\,\frac{s_{ij}}{s_{ii}} \qquad i,j=<1,\ldots,n>,\ s_{ii}\neq 0 \tag{8}$$

s_{ii}, s_{ij} - algebraic supplements of separate elements of a transfer matrix of controlled plant $\boldsymbol{G_S}(s)$

S_{ij} - separate members of a transfer matrix of controlled plant $\boldsymbol{G_S}(s)$

- Ensuring of autonomy of control loop via **binding members** of transfer matrix of controller $\boldsymbol{G_R}$(s). Binding members are determined from equation (5).
- Ensuring of invariance control loop via **correction members** *KC* by using *n* SISO branched control loops with measuring of disturbance variables. Correction members are determined from equation (7).

3. SIMULATION VERIFICATION

3.1 MIMO controlled system

Typical example of MIMO controlled plant is a steam turbine. In this case is considered the turbine with two controlled withdrawals which drives electric generator supplying determined part of electric network (it operates without phasing into power network). The turbine represent three-variable control loop. The scheme of three-variable control loop of steam turbine is presented on the following Fig. 3.

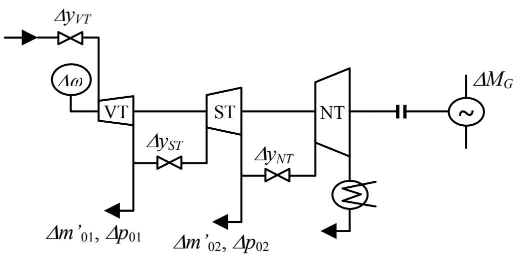

Fig. 3. Three-variable control loop of steam turbine

Legend: ΔM_G - change of electric load of turbo-generator, $\Delta\omega$ - change of angular speed of turbo-generator, $\Delta m'_{01}$, $\Delta m'_{02}$ - change of mass flow of withdrawn steam, Δp_{01}, Δp_{02} - change of steam pressure in corresponding withdrawals, Δy_{VT}, Δy_{ST}, Δy_{NT} - change of opening position of control valves of high-pressure, medium-pressure and low-pressure part of turbine.

It is consider that controlled variables y are $\Delta\omega$, Δp_{01}, Δp_{02}, manipulated variables u are Δy_{VT}, Δy_{ST}, $\Delta\ y_{NT}$ and disturbance variables v are ΔM_G, $\Delta m'_{01}$, $\Delta m'_{02}$.

3.2 Mathematical model of three-variable control loop

Resulting differential equations for creating mathematical model of the plant were gained already after deriving and using linearization from the project OTROKOVICE elaborated by the firm ALSTOM Power (ALSTOM Power, 1998). Differential equations were re-written into other form by introducing relative values, with regard to starting steady state-operational, i.e. to calculated point, at which relation of values can be generally written in the form $\varphi_X = \Delta X/(X)_0$. The Laplace transform of an output (controlled) variable then was given by the following relation

$$\begin{bmatrix} \varphi_\omega \\ \varphi_{p_{01}} \\ \varphi_{p_{02}} \end{bmatrix} = \boldsymbol{G}_S(s) \begin{bmatrix} \varphi_{y_{VT}} \\ \varphi_{y_{ST}} \\ \varphi_{y_{NT}} \end{bmatrix} + \boldsymbol{G}_{SV}(s) \begin{bmatrix} \varphi_{M_G} \\ \varphi_{m'_{01}} \\ \varphi_{m'_{02}} \end{bmatrix} \tag{9}$$

where

$$\boldsymbol{G}_S(s) = \begin{bmatrix} \frac{0.73s^2 + 1.59s + 1.11}{12.3s^3 + 21.8s^2 + 10.7s + 1} & \frac{0.46s^2 + 0.74s + 0.09}{12.3s^3 + 21.8s^2 + 10.7s + 1} & \frac{0.32s^2 + 0.32s + 0.04}{12.3s^3 + 21.8s^2 + 10.7s + 1} \\ \frac{1.68s + 1.31}{1.51s^2 + 2.48s + 1} & \frac{-1.25s - 0.96}{1.51s^2 + 2.48s + 1} & \frac{-0.01}{1.51s^2 + 2.48s + 1} \\ \frac{1.76}{1.51s^2 + 2.48s + 1} & \frac{1.56s + 0.040}{1.51s^2 + 2.48s + 1} & \frac{-1.12s - 0.97}{1.51s^2 + 2.48s + 1} \end{bmatrix} \tag{10}$$

$$\boldsymbol{G}_{SV}(s) = \begin{bmatrix} \frac{-1.51s^2 - 2.48s - 1}{12.3s^3 + 21.8s^2 + 10.7s + 1} & \frac{-0.09s - 0.15}{12.3s^3 + 21.8s^2 + 10.7s + 1} & \frac{-0.09s - 0.08}{12.3s^3 + 21.8s^2 + 10.7s + 1} \\ 0 & \frac{-0.40s - 031}{1.51s^2 + 2.48s + 1} & \frac{-0,005}{1.51s^2 + 2.48s + 1} \\ 0 & \frac{-0.420}{1.51s^2 + 2.48s + 1} & \frac{-0.501s - 0.432}{1.51s^2 + 2.48s + 1} \end{bmatrix} \tag{11}$$

3.3 Synthesis of three-variable control loop of a steam turbine

The principal described in the paragraph 2.2 is used at solution of synthesis of the three-variable control loop. First transfers of main controllers R_{11}, R_{22}, R_{33} are determined for modified diagonal transfer functions $S_{11,x}$, $S_{22,x}$ a $S_{33,x}$ (equation (8)) then autonomy of control loop by using relation (5) is being solved and in the end fulfilment of the condition of invariance (approximate invariance) of control loop is ensured by using equation (7). At design of parameters of main controllers the following SISO methods of synthesis were used: Ziegler Nichols step response method (Balate, 2004), method of desired model (Wagnerova & Minar, 2000), polynomial method of synthesis for 1DOF configuration (Prokop et al., 2006). In the next part of the paper, chosen method for design of parameters of main controllers, is used, i.e. a method of desired model. Transfer matrixe of controllers $\boldsymbol{G_R}(s)$ with utilization of a chosen method and transfer matrix of correction members $\boldsymbol{G_{KC}}(s)$ are given by the equations (12) and (13).

$$\boldsymbol{G}_R(s) = \begin{bmatrix} \frac{1.06s + 0.13}{s} & \frac{0.056s + 0.007}{s} & \frac{0.028s + 0.036}{s} \\ \frac{1.42s + 0.17}{s} & \frac{-0.045s - 0.095}{s} & \frac{0.037s + 0.006}{s} \\ \frac{1.99s + 0.24}{s} & \frac{-0.063s + 0.009}{s} & \frac{-0.12s - 0.12}{s} \end{bmatrix} \tag{12}$$

$$\boldsymbol{G}_{KC}(s) = \begin{bmatrix} \frac{-2.06s^2 - 3.39s - 1.37}{s^2 + 2.18s + 1.52} & 0 & 0 \\ 0 & \frac{0.32s + 0.25}{s + 0.77} & 0 \\ 0 & 0 & 0.45 \end{bmatrix} \tag{13}$$

3.4 Simulation results

Simulation course of three-variable control loop of a steam turbine with utilization of chosen SISO synthesis method is presented on the Fig. 4.

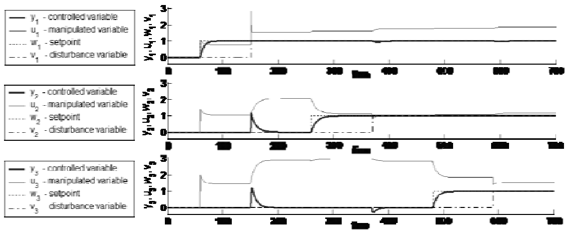

Fig. 4. Simulation course of control loop with utilization of method of desired model

Variables on Fig. 4 correspond to variables existing in the three-variable control loop of steam turbine, i.e.

$y_1 \to \varphi_\omega,\ y_2 \to \varphi_{p_{01}}, y_3 \to \varphi_{p_{02}}, v_1 \to \varphi_{M_G}, v_2 \to \varphi_{m'_{01}}, v_3 \to \varphi_{m'_{02}},$

$u_1 \to \varphi_{y_{VT}}, u_2 \to \varphi_{y_{ST}}, u_3 \to \varphi_{y_{NT}}$

3.5 Evalutation of simulation experiment

It is obvious from simulation course of control process presented on Fig. 4 and from other simulation experiments (Navratil & Balate, 2007) that the condition of autonomy and invariance was fulfilled. Fulfilment of the condition of autonomy was ensured via binding members R_{ij}, which are aside-from-diagonal elements of transfer matrix of controller $\boldsymbol{G_R}(s)$. Fulfilment of the condition of invariance was ensured by means of correction members KC_{ii} which are considered for elimination of influence of dominant disturbance variables by means of using analogy of SISO branched control loop with measurement of disturbance v.

5. CONCLUSION

In this paper one of possible methods to control of multivariable control loop was described. Simulation verification of proposed method of control was presented on three-variable control loop of steam turbine. At first the method deals with setting-up of main (diagonal) controllers then determination of binding members for ensuring autonomy and in the end determination of correction members for ensuring invariance.

6. ACKNOWLEDGEMENT:

This work was supported by the Ministry of Education of the Czech Republic under grant No. MSM 7088352102.

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