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POLYNOMIAL APPROACH TO DISCRETE-TIME ADAPTIVE CONTROL: SOFTWARE IMPLEMENTATION FOR INDUSTRIAL APPLICATION

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Abstract: *The main aim of this paper is to present a preliminary industrial software implementation of selected discrete-time adaptive control algorithms into the Matlab and Pascal environment. The motivation and basic conditions of this application have been based on real technical task of a manufacturer of aluminium-based rolled products and packaging materials. The applied methods include a polynomial approach to discrete-time control design and recursive least-squares identification algorithm LDDIF.*
Key words: *adaptive control, discrete-Time control, Polynomial approach, implementation*

1. INTRODUCTION

Real control of industrial processes is almost always burden with various perturbations, disturbances and changes in process parameters or dynamics due to varying operational conditions, plant properties themselves, etc. Furthermore, an acceptable a priori mathematical model does not have to be known. In spite of it, such processes have to be controlled.

A possible solution to this task represents an area of control theory know as adaptive control or more specifically usage of self-tuning controllers (Bobál *et al.*, 1999). Main idea consists in modification of control law according to the changing plant parameters obtained via recursive identification. Its advantage is some kind of "intelligent" behaviour, but on the other hand these regulators are quite complex and not easily applicable.

This contribution deals with software implementation of discrete-time adaptive control algorithms into the Matlab and Pascal environment. The work was motivated by co-operation with a manufacturer of aluminium-based rolled products and packaging materials. His project has supposed the use of discrete-time adaptive controller, plant model with "a2b3" structure and final implementation in Borland Pascal (because of integration into the existing system). However the paper presents not only derived relations applicable to Pascal environment but also preliminary program for simulative purposes and testing created under Matlab.

2. DISCRETE-TIME POLYNOMIAL SYNTHESIS

The classical control configuration with one degree of freedom as shown in fig. 1 is assumed.

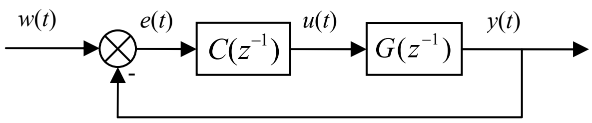

Fig. 1. Basic closed control loop

According to the project requirements a controlled plant is supposed to has an "a2b3" structure, i.e. its transfer function is:

$$G(z^{-1}) = \frac{b(z^{-1})}{a(z^{-1})} = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{1}$$

A suitable controller which ensures stabilization of the control circuit and reference tracking can be obtained via solution of Diophantine equation (Kučera, 1993):

$$a(z^{-1}) f(z^{-1}) p(z^{-1}) + b(z^{-1}) q(z^{-1}) = m(z^{-1}) \tag{2}$$

where a, b are from the controlled system (1), p, q from discrete-time controller:

$$C(z^{-1}) = \frac{q(z^{-1})}{f(z^{-1}) p(z^{-1})} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{f(z^{-1})\left(p_0 + p_1 z^{-1} + p_2 z^{-2}\right)} \tag{3}$$

and where f is the denominator of stepwise reference signal:

$$W(z^{-1}) = \frac{1}{f(z^{-1})} = \frac{1}{1 - z^{-1}} \tag{4}$$

Moreover, m is a stable polynomial of appropriate order. Thus the equation (2) takes here the specific form:

$$\begin{aligned} &\left(1 + a_1 z^{-1} + a_2 z^{-2}\right)\left(1 - z^{-1}\right)\left(p_0 + p_1 z^{-1} + p_2 z^{-2}\right) + \cdots \\ &\cdots \left(b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}\right)\left(q_0 + q_1 z^{-1} + q_2 z^{-2}\right) = \\ &= m_0 + m_1 z^{-1} + m_2 z^{-2} + m_3 z^{-3} + m_4 z^{-4} + m_5 z^{-5} \end{aligned} \tag{5}$$

The relation (5) leads to the system of equations in a matrix form:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ a_1 - 1 & 1 & 0 & b_1 & 0 & 0 \\ a_2 - a_1 & a_1 - 1 & 1 & b_2 & b_1 & 0 \\ -a_2 & a_2 - a_1 & a_1 - 1 & b_3 & b_2 & b_1 \\ 0 & -a_2 & a_2 - a_1 & 0 & b_3 & b_2 \\ 0 & 0 & -a_2 & 0 & 0 & b_3 \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ q_0 \\ q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} m_0 \\ m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \end{pmatrix} \tag{6}$$

which would be an easy task for solving in many software packages. However, the submitter required the final implementation of control algorithms in Borland Pascal and so the analytical solution of the set (6) had to be derived. Thus, the applicable controller parameters are:

$$\begin{aligned} q_0 &= h_{13} / h_{12} \\ q_1 &= h_{14} / h_{12} \\ q_2 &= h_{15} / h_{12} \\ p_0 &= m_0 \\ p_1 &= h_2 - b_1 q_0 \\ p_2 &= \left(q_2 b_3 - m_5\right) / a_2 \end{aligned} \tag{7}$$

where

$$
\begin{aligned}
&h_1 = (a_2 - a_1) b_3 + b_2 a_2; \qquad h_2 = m_1 + (1 - a_1) m_0 \\
&h_3 = \left[a_2 m_4 + a_2^2 h_2 + (a_2 - a_1) m_5 \right] / h_1 \\
&h_4 = \left(a_2^2 b_1 \right) / h_1; \qquad h_5 = (a_2 b_3) / h_1 \\
&h_6 = m_0 (a_2 - a_1) + h_2 a_1 - h_2; \qquad h_7 = -b_1 a_1 + b_1 + b_2 \\
&h_8 = m_2 - h_6 + m_5 / a_2; \qquad h_9 = -b_1 a_2 + b_1 a_1 + b_3 \\
&h_{10} = (b_3 a_1) / a_2 - b_3 / a_2 + b_1 \\
&h_{11} = m_3 + m_0 a_2 - h_2 a_2 + h_2 a_1 + (m_5 a_1) / a_2 - m_5 / a_2 \\
&h_{12} = h_4 b_1 h_{10} + h_7 b_2 + h_9 h_5 (b_3 / a_2) - b_1 h_9 - \cdots \\
&\qquad \cdots (b_3 / a_2) b_2 h_4 - h_{10} h_5 h_7 \\
&h_{13} = h_3 b_1 h_{10} + h_8 b_2 + h_{11} h_5 (b_3 / a_2) - b_1 h_{11} - \cdots \\
&\qquad \cdots (b_3 / a_2) b_2 h_3 - h_{10} h_5 h_8 \\
&h_{14} = h_4 h_8 h_{10} + h_7 h_{11} + h_9 h_3 (b_3 / a_2) - h_8 h_9 - \cdots \\
&\qquad \cdots (b_3 / a_2) h_{11} h_4 - h_{10} h_3 h_7 \\
&h_{15} = h_4 b_1 h_{11} + h_7 b_2 h_3 + h_9 h_5 h_8 - h_3 b_1 h_9 - h_8 b_2 h_4 - h_{11} h_5 h_7
\end{aligned}
\tag{8}
$$

The coefficients of the polynomial m can be used for controller tuning and thus for influencing the closed-loop control behaviour.

The programmable control law which corresponds to the controller (3) and which generates the control signal $u(k)$ can be written as:

$$
u(k) = \left[\begin{array}{l} (p_0 - p_1) u(k-1) + (p_1 - p_2) u(k-2) + \cdots \\ \cdots p_2 u(k-3) + q_0 e(k) + q_1 e(k-1) + q_2 e(k-2) \end{array} \right] \Big/ p_0 \tag{9}
$$

3. RECURSIVE IDENTIFICATION ALGORITHM

A LDDIF routine was used as identification technique. It is recursive least-squares algorithm with exponential and directional forgetting (Kulhavý & Kárný, 1984). Moreover, the corrections influencing the covariance matrix of the estimated parameters by adding some multiple of identity matrix, suggested in (Bittanti *et al.*, 1990), are implemented to improve the tracking performance.

The algorithm can be described by equations (Čirka *et al.*, 2006):

$$
\begin{aligned}
&\varepsilon(k) = \boldsymbol{y}(k) - \boldsymbol{\Phi}(k)^T \boldsymbol{\theta}(k-1) \\
&r(k) = \boldsymbol{\Phi}(k)^T \boldsymbol{P}(k-1) \boldsymbol{\Phi}(k) \\
&\boldsymbol{\kappa}(k) = \frac{\boldsymbol{P}(k-1)\boldsymbol{\Phi}(k)}{1 + r(k)} \\
&\beta(k) = \begin{cases} \varphi - \dfrac{1-\varphi}{r(k)} & \text{if } r(k) > 0 \\ 1 & \text{if } r(k) > 0 \end{cases} \\
&\boldsymbol{P}(k) = \boldsymbol{P}(k-1) - \frac{\boldsymbol{P}(k-1)\boldsymbol{\Phi}(k)\boldsymbol{\Phi}(k)^T \boldsymbol{P}(k-1)}{\beta(k)^{-1} + r(k)} + \delta \boldsymbol{I} \\
&\boldsymbol{\theta}(k) = \boldsymbol{\theta}(k-1) + \boldsymbol{\kappa}(k) \varepsilon(k)
\end{aligned}
\tag{10}
$$

where $\boldsymbol{\Phi}(k) = \left[-y(k-1) \;\; -y(k-2) \;\; u(k-1) \;\; u(k-2) \;\; u(k-3) \right]$ is observation vector, $\theta(k) = \left[a_1(k) \;\; a_2(k) \;\; b_1(k) \;\; b_2(k) \;\; b_3(k) \right]$ is vector of parameters and φ is exponential forgetting. The initial values for the algorithm are usually $\varphi = 0.985$, $\boldsymbol{P}(0) = 10^6 \boldsymbol{I}$ and $\delta = 0.01$.

4. SOFTWARE IMPLEMENTATION

As it was adumbrated before, Borland Pascal had to be used for final implementation in real industrial conditions. However, several preliminary tests, algorithm verifications and simulations were done in Matlab environment due to better convenience for these purposes. As a result, a simple program has been created. Its main window is shown in fig. 2.

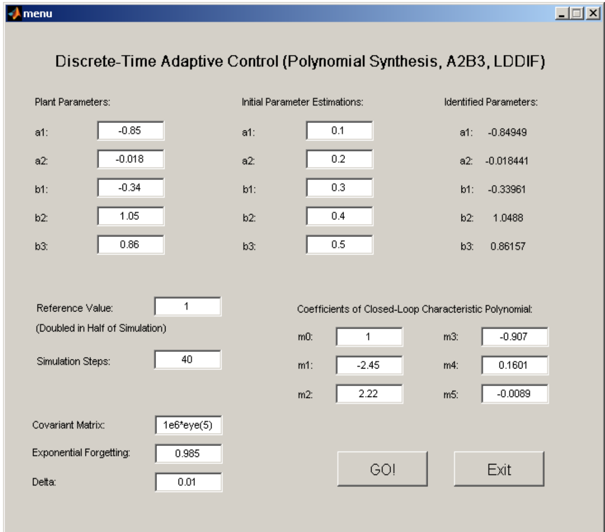

Fig. 2. Main window of the preliminary simulation program in Matlab

5. CONCLUSION

This contribution has been focused on preliminary software implementation of digital self-tuning controllers into the Matlab (for simulative and testing purposes) and Pascal (for real application) environment. The motivation to this task as well as basic conditions and restrictions have been based on technical assignment of a manufacturer of aluminium-based products. The applied techniques have comprised a polynomial approach to discrete-time control design and recursive least-squares identification algorithm LDDIF. The future work should eventually lead to complete real-time industrial application.

6. ACKNOWLEDGEMENT

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7. REFERENCES

Bittanti, S.; Bolzern, P. & Campi, M. (1990). Exponential Convergence of a Modified Directional Forgetting Identification Algorithm. *Systems & Control Letters*, Vol. 14, No. 2, pp. 131-137, ISSN 0167-6911

Bobál, V.; Böhm, J.; Prokop, R. & Fessl, J. (1999). *Praktické aspekty samočinně se nastavujících regulátorů: algoritmy a implementace*. (*Practical Aspects of Self-Tuning Controllers: Algorithms and Implementation*). Brno University of Technology, VUTIUM, ISBN 80-214-1299-2, Brno, Czech Republic. (In Czech)

Čirka, Ľ.; Fikar, M. & Petruš, P. (2006). IDTOOL 4.0 – A Dynamical System Identification Toolbox for Matlab/ Simulink, *Proceedings of Technical Computing Prague 2006*, Prague, Czech Republic, 2006

Kučera, V. (1993). Diophantine Equations in Control – A Survey. *Automatica*, Vol. 29, No. 6, pp. 1361-1375, ISSN 0005-1098

Kulhavý, R. & Kárný, M. (1984). Tracking of Slowly Varying Parameters by Directional Forgetting. *Proceedings of the 9th IFAC World Congress*, pp. 79-83, Budapest, Hungary, 1984