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Research Article

Robust Stability of Fractional Order Time-Delay Control Systems: A Graphical Approach

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The paper deals with a graphical approach to investigation of robust stability for a feedback control loop with an uncertain fractional order time-delay plant and integer order or fractional order controller. Robust stability analysis is based on plotting the value sets for a suitable range of frequencies and subsequent verification of the zero exclusion condition fulfillment. The computational examples present the typical shapes of the value sets of a family of closed-loop characteristic quasipolynomials for a fractional order plant with uncertain gain, time constant, or time-delay term, respectively, and also for combined cases. Moreover, the practically oriented example focused on robust stability analysis of main irrigation canal pool controlled by either classical integer order PID or fractional order PI controller is included as well.

1. Introduction

Recently, the fractional order calculus (FOC) and its engineering applications represent attractive research field with rapidly growing amount of related scientific works. This progress is understandable since the use of differentiation and integration under an arbitrary real or even complex number of the operations provides efficient tool for many real-life problems and since the knowledge of suitable and relatively comprehensible mathematical instruments for fractional order issues has increased lately. The principal sources for studying the FOC are, for example, the monographs [1–3] and possibly also [4] or [5]. The FOC has already been useful in areas such as bioengineering, viscoelasticity, electronics, robotics, control theory, and signal processing [6, 7]. The examples of several useful control-oriented works can be seen in [8–12]. Obviously, the FOC has influenced also analysis and control of time-delay systems which represent usually complicated but relatively frequent controlled objects [13–17].

Models with parametric uncertainty are popular and effective way to uncertainty modelling and consequently to description of too complicated, nonlinear, or varying real-life systems by means of linear models. In such systems, the structure (model order) is supposed or known, but the parameters are bounded somehow. Typically, they lie within given intervals. One of the related principal tasks consists in robust stability analysis, that is, in investigation of keeping the stability under all possible variations of uncertain parameters. Some authors have already tried to combine the issue of robust stability of systems affected by parametric uncertainty with fractional order systems, for example, [18–28].

This paper is focused on a graphical approach to robust stability analysis and especially on its application to fractional order time-delay control systems. More specifically, the control loop studied in the computational examples consists of a fractional order time-delay plant with uncertain parameters and standard integer order PID controller. The robust stability is tested via plotting the value sets of a closed-loop characteristic quasipolynomial and application of the zero exclusion condition. The presented examples include the typical shapes of the value sets for a fractional order controlled system with uncertain gain, time constant, or time-delay term, respectively, and then also for the case of all uncertain parameters together. Moreover, the final process-control-oriented example deals with robust stability analysis for main irrigation canal pool controlled by either classical

PID or fractional order PI controller. This paper is the significantly extended version of the conference contribution [29].

The paper is organized as follows. In Section 2, basic theoretical background and description of fractional order systems are provided. Section 3 then presents the robust stability analysis for integer order and fractional order systems with parametric uncertainty with especial emphasis on the value set concept and the zero exclusion condition. Next, a number of computational examples and visualizations of the value sets for closed loop containing a fractional order time-delay plant with various uncertain parameters are shown in the extensive Section 4. Further, Section 5 contains more specific and practically oriented example motivated by control of main irrigation canals with variable parameters. And finally, Section 6 offers some concluding remarks.

2. Fractional Order Systems

The FOC is grounded in generalization of differentiation and integration to an arbitrary (rational, irrational, or even complex) order. This generalization has resulted in the introduction of basic continuous differintegral operator [1, 2, 4, 6]:

$$ {}_aD_t^\alpha = \begin{cases} \dfrac{d^\alpha}{dt^\alpha} & \operatorname{Re}\alpha > 0 \\ 1 & \operatorname{Re}\alpha = 0 \\ \displaystyle\int_a^t (d\tau)^{-\alpha} & \operatorname{Re}\alpha < 0, \end{cases} \quad (1) $$

where α is the order of the differintegration (ordinarily $\alpha \in \mathbb{R}$) and a is a constant related to initial conditions. The differintegral can be defined in various ways. The three most common ones are Riemann-Liouville, Grünwald-Letnikov, and Caputo definitions.

The Laplace transform of the differintegral is given by [4, 9]

$$ \begin{aligned} L\left\{ {}_aD_t^\alpha f(t) \right\} &= \int_0^\infty e^{-st} {}_0D_t^\alpha f(t)\, dt \\ &= s^\alpha F(s) \\ &\quad - \sum_{m=0}^{n-1} s^m (-1)^j {}_0D_t^{\alpha-m-1} f(t)\Big|_{t=0}, \end{aligned} \quad (2) $$

where integer n lies within $(n-1 < \alpha \le n)$.

The (time-delay-free) fractional order transfer function can be written as [3, 5]

$$ G(s) = \frac{B(s^{\beta_k})}{A(s^{\alpha_k})} = \frac{b_m s^{\beta_m} + b_{m-1}s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1}s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}}, \quad (3) $$

where a_k with $(k = 0, \ldots, n)$ and b_k with $(k = 0, \ldots, m)$ denote constants and α_k with $(k = 0, \ldots, n)$ and β_k with $(k = 0, \ldots, m)$ are arbitrary real numbers. According to [4, 5], one can assume inequalities $\alpha_n > \alpha_{n-1} > \cdots > \alpha_0$ and $\beta_m > \beta_{m-1} > \cdots > \beta_0$ without loss of generality. In this paper, the controlled time-delay system is supposed generally as

$$ G(s) = \frac{B(s^{\beta_k})}{A(s^{\alpha_k})} e^{-\Theta s}. \quad (4) $$

3. Robust Stability Analysis under Parametric Uncertainty

The stability of the closed-loop system will be tested via stability of its characteristic polynomial (or quasipolynomial in the case of this paper).

The continuous-time fractional order uncertain polynomial can have the form

$$ \begin{aligned} p(s,q) &= \rho_n(q) s^{\alpha_n} + \rho_{n-1}(q) s^{\alpha_{n-1}} + \cdots + \rho_1(q) s^{\alpha_1} \\ &\quad + \rho_0(q) s^{\alpha_0}, \end{aligned} \quad (5) $$

where q is the vector of uncertainty and ρ_k for $k = 0, 1, 2 \ldots, n$ are coefficient functions. Besides, the characteristic quasipolynomial (for closed control loop with time-delay plant) would contain the term $e^{-\Theta s}$.

Then, the family of polynomials is [30]

$$ P = \{ p(\cdot, q) : q \in Q \}, \quad (6) $$

where Q is the uncertainty bounding set (frequently, it is a multidimensional box).

The family of polynomials (6) is robustly stable if and only if $p(s,q)$ is stable for all $q \in Q$. The choice of technique for investigation of robust stability depends primarily on the structure of uncertainty. Generally, the higher level of relation among coefficients entails more complex robust stability analysis which requires more sophisticated tools. However, one graphical method seems to be unique from the viewpoint of its universality and applicability. It is based on combination of the value set concept and the zero exclusion condition [30]. It can be applied for a wide range of uncertainty structures, from the simplest to the very complicated ones. Moreover, it is applicable also for various regions of stability (robust D-stability). The detailed information on parametric uncertainty and robust stability analysis as well as examples of the typical value sets can be found in [30] and subsequently, for example, in [31, 32]. And finally, [18–21] have extended the idea of the value set concept also to fractional order uncertain polynomials.

Under assumption of a family of polynomials (6), the value set at frequency $\omega \in \mathbb{R}$ is given by [30]

$$ p(j\omega, Q) = \{ p(j\omega, q) : q \in Q \}. \quad (7) $$

It means that $p(j\omega, Q)$ is the image of Q under $p(j\omega, \cdot)$. Practical construction of the value sets can be accomplished by substituting s for $j\omega$, fixing ω, and letting the vector of uncertain parameters q range over the set Q.

The zero exclusion condition for Hurwitz stability of family of continuous-time polynomials (6) is defined as follows [30]: assume invariant degree of polynomials in the family, pathwise connected uncertainty bounding set Q, continuous coefficient functions $\rho_k(q)$ for $k = 0, 1, 2, \ldots, n$, and at least one stable member $p(s, q^0)$. Then the family P is robustly stable if and only if the complex plane origin is excluded from the value set $p(j\omega, Q)$ at all frequencies $\omega \ge 0$; that is, P is robustly stable if and only if

$$ 0 \notin p(j\omega, Q) \quad \forall \omega \ge 0. \quad (8) $$

Authors of [18–21] construct the value sets of the fractional order families of polynomials mainly on the basis of the fact that the fractional power of $j\omega$ can be written as

$$(j\omega)^{\alpha} = \omega^{\alpha}\left(\cos\frac{\pi}{2}\alpha + j\sin\frac{\pi}{2}\alpha\right) \tag{9}$$

and on the consequent analysis of vertices and exposed edges.

In this work, the value sets are plotted for quasipolynomials (closed-loop characteristic quasipolynomials of the feedback circuits with the uncertain time-delay fractional order plant and fixed integer order or fractional order controller) and their visualization is based on sampling the uncertain parameters and on computation of partial points of the value sets for a considered frequency range. Thanks to the applied sampling (brute-force) method, the value sets of quasipolynomials can be easily computed and consequently the robust stability can be investigated with the assistance of standard zero exclusion condition. The technique itself should be clear from the following examples.

4. Computational Examples: Typical Shapes of Value Sets

Consider a fractional order time-delay plant given by

$$G(s, K, T, \Theta) = \frac{K}{Ts^{\alpha} + 1}e^{-\Theta s}, \tag{10}$$

where K is a gain, T stands for a time constant, α is a real number representing the fractional order of the dynamics, and Θ is a time-delay term. One or more of the parameters K, T, and Θ are uncertain and they can vary within given intervals.

More specifically, the controlled system is described, for example, as

$$G(s, K, T, \Theta) = \frac{K}{Ts^{0.75} + 1}e^{-\Theta s}, \tag{11}$$

where either one of the parameters is uncertain:

$$K = [7, 13]; \quad T = 3; \quad \Theta = 2, \tag{12}$$

$$K = 10; \quad T = [2, 4]; \quad \Theta = 2, \tag{13}$$

$$K = 10; \quad T = 3; \quad \Theta = [1.5, 2.5] \tag{14}$$

or all of them can lie within supposed bounds:

$$K = [7, 13]; \quad T = [2, 4]; \quad \Theta = [1.5, 2.5]. \tag{15}$$

In all cases, the nominal system used for the controller design is assumed with the fixed (average) values:

$$G_N(s) = \frac{10}{3s^{0.75} + 1}e^{-2s}. \tag{16}$$

The PID controller for this plant could be obtained, for example, with the assistance of the FOMCON Toolbox for MATLAB [33, 34] and its routine "iopid_tune." More specifically, the Oustaloup filter based [35] approximation leads to the integer order model:

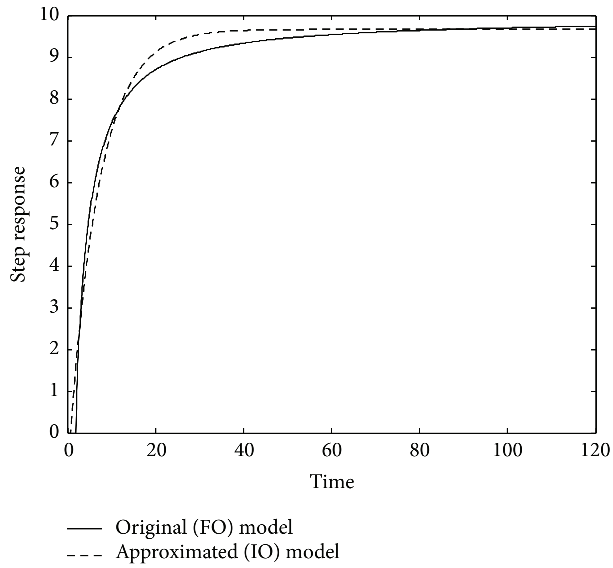

Figure 1: Comparison of step responses of original (FO) model (16) and approximated (IO) model (17).

$$G_A(s) = \frac{9.66313}{6.75338s + 1}e^{-0.736803s}. \tag{17}$$

The selected controller for this plant has the form

$$C(s) = K_p + \frac{K_i}{s} + K_d s = 0.1 + \frac{0.05}{s} + 0.01s. \tag{18}$$

More information on integer order approximations of fractional order systems can be found, for example, in [36]. The comparison between step responses of the fractional order (FO) model and its integer order (IO) approximation can be seen in Figure 1. It is still obtained through the FOMCON Toolbox.

The control responses for the loops with controller (18) and original nominal (FO) model (16) or approximated (IO) model (17), respectively, are compared in Figure 2.

Nevertheless, the approximation was done only for the sake of IO controller choice. The robust stability of the closed-loop control system will be investigated by means of the family of its characteristic quasipolynomials, which contains the true FO model (11):

$$\begin{aligned} p_{\mathrm{cl}}(s, K, T, \Theta) = {} & \left(Ts^{0.75} + 1\right)s \\ & + Ke^{-\Theta s}\left(K_d s^2 + K_p s + K_i\right), \end{aligned} \tag{19}$$

where K_p, K_i, and K_d are fixed PID controller parameters from (18) while one or more of the coefficients K, T, and Θ of plant (11) can vary according to (12)–(15).

First, only the gain is supposed to be uncertain while the time constant and time-delay term remain fixed; that is, scenario (12) holds true.

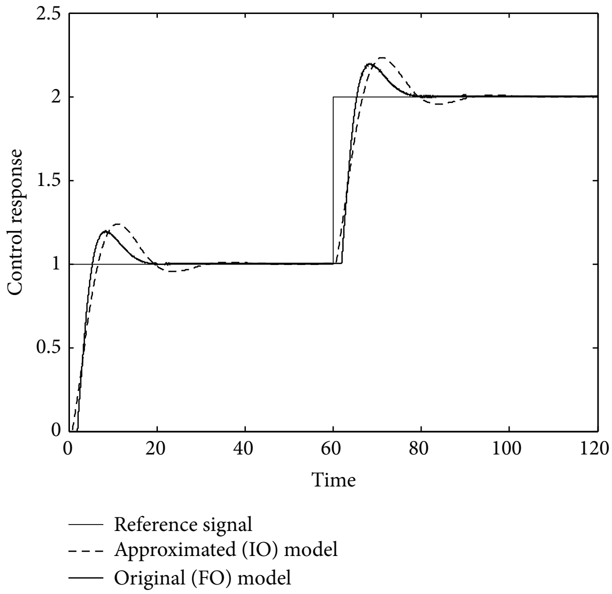

FIGURE 2: Comparison of control responses of original (FO) model (16) and approximated (IO) model (17).

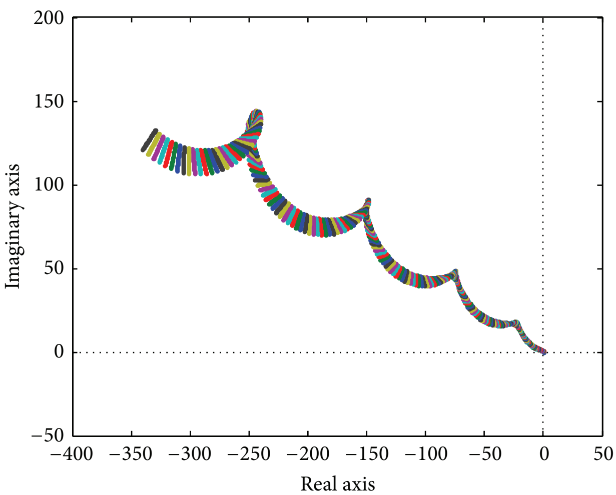

FIGURE 3: Value sets for controller (18) and plant (11) with parameters (12).

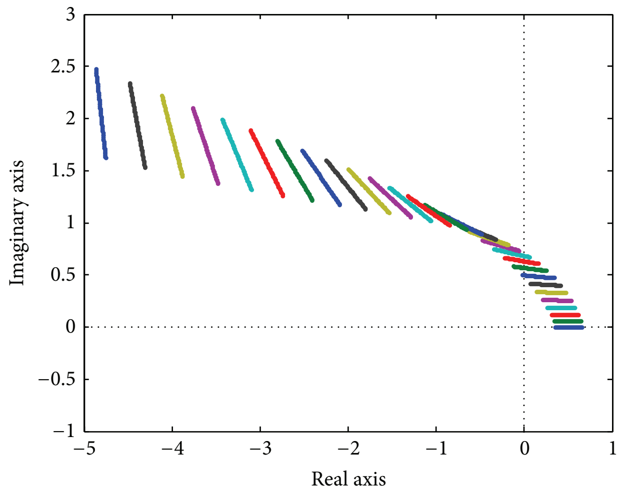

FIGURE 4: Value sets for controller (18) and plant (11) with parameters (12)—zoomed version.

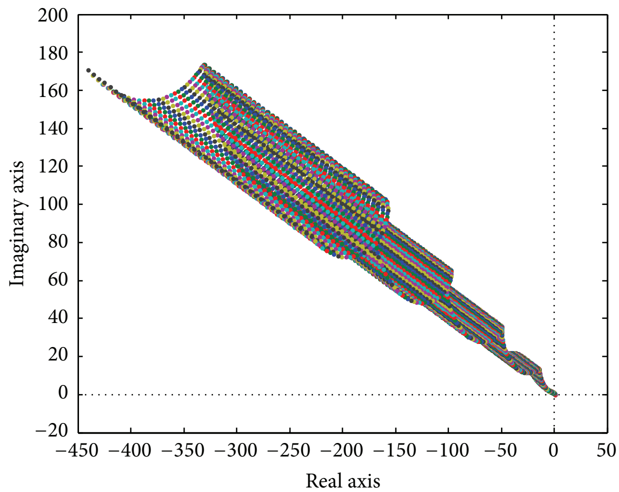

FIGURE 5: Value sets for controller (18) and plant (11) with parameters (13).

The straight-line value sets computed for the corresponding family of closed-loop characteristic quasipolynomials for the range of frequencies from 0 to 15 with the step 0.05 are depicted in Figure 3. At each frequency, K is sampled within given interval with the step 0.1 (i.e., each line consists of 61 points). Then, the zoomed version for better view of the situation near the origin of the complex plane is shown in Figure 4. As can be seen, the zero point is excluded from the value sets. Thus, because the family contains at least one stable member (see Figure 2) and the zero is excluded, the family is robustly stable. In other words, the closed-loop control system with the fractional order uncertain plant (11) with (12) and with fixed PID controller (18) remains stable for all possible values of gain from supposed interval.

Now, the time constant is going to be the only uncertain parameter according to (13). Figure 5 shows the resulting value sets for the family of closed-loop characteristic quasipolynomials (in the same range of frequencies as in the previous case). Again, T is sampled with the step 0.05 and consequently every straight-line value set consists of only 41 points. The closer look to the complex plane origin is provided by Figure 6, which clearly indicates that the zero point is not included in the value sets. Analogically to the previous example, the family can be considered as the robustly stable one.

Next simulation scenario is given by (14); that is, the time-delay term is the uncertain parameter. The respective value sets for again $\omega = 0 : 0.05 : 15$ are shown in Figure 7. Each value set is not a straight-line now but more complex single-parameter curve. Time-delay term is sampled according to $\Theta = 1.5 : 0.02 : 2.5$, which gives the value set as a curve

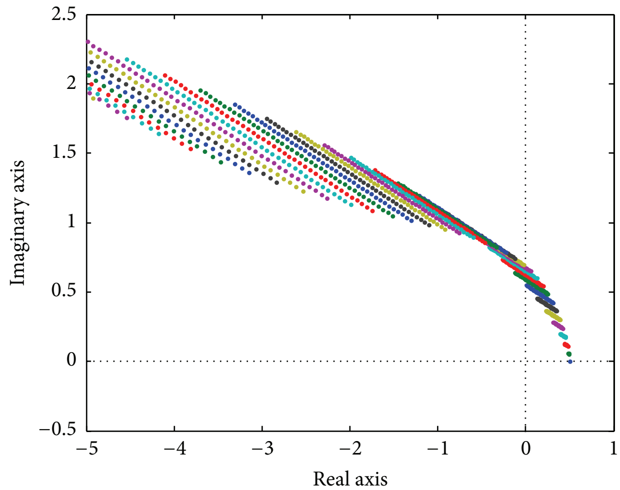

Figure 6: Value sets for controller (18) and plant (11) with parameters (13)—zoomed version.

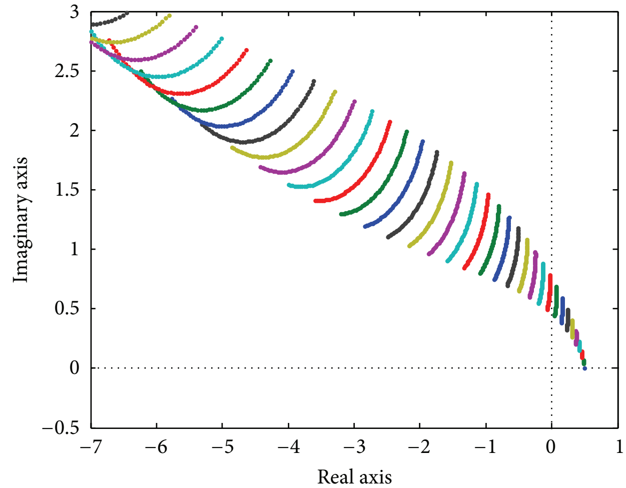

Figure 8: Value sets for controller (18) and plant (11) with parameters (14)—zoomed version.

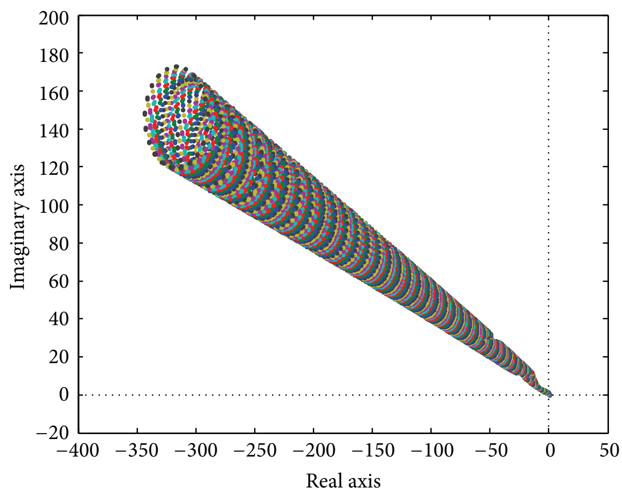

Figure 7: Value sets for controller (18) and plant (11) with parameters (14).

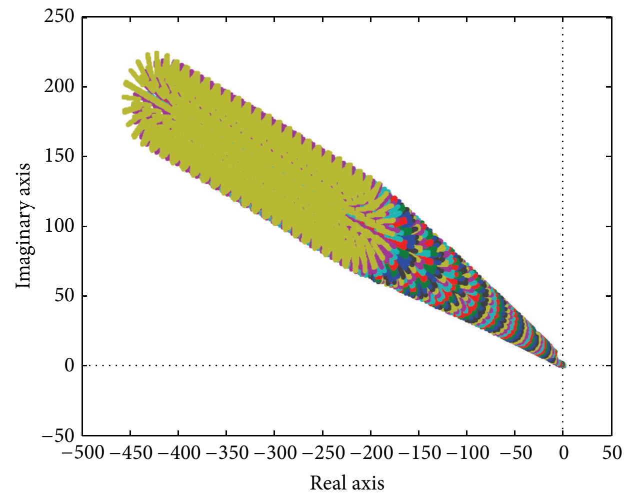

Figure 9: Value sets for controller (18) and plant (11) with parameters (15).

plotted via 51 points. The detailed view in Figure 8 reveals that the closed-loop system is robustly stable also in this case.

Finally, the controlled plant with all three varying parameters is assumed—see (15)—and its value sets are plotted in Figure 9. The frequency and the plant parameters are sampled as follows:

$$\begin{aligned} \omega &= 0 : 0.2 : 15 \\ K &= 7 : 0.2 : 13 \\ T &= 2 : 0.1 : 4 \\ \Theta &= 1.5 : 0.04 : 2.5. \end{aligned} \tag{20}$$

The family definitely contains a stable member and the zero point is excluded from the value sets (as can be seen from the zoomed Figure 10 where the step of frequency is lowered to 0.1), so the family and the whole closed-loop control system are robustly stable even in this event.

In addition to all robustly stable cases shown in the previous parts, one can very easily obtain the family which is robustly unstable. For example, assume the uncertain gain case (12) with a different PID controller:

$$C(s) = K_p + \frac{K_i}{s} + K_d s = 0.3 + \frac{0.15}{s} + 0.03s \tag{21}$$

which results in the value sets in Figure 11 and its zoomed version in Figure 12 (for $\omega = 0 : 0.05 : 15$). Now, the value sets include the complex plane origin and thus the family of closed-loop characteristic quasipolynomials is not robustly stable (i.e., the system would be unstable for some possible values of K).

Nevertheless, even if the stability test using the zero exclusion condition is visually very simple, one has to be careful

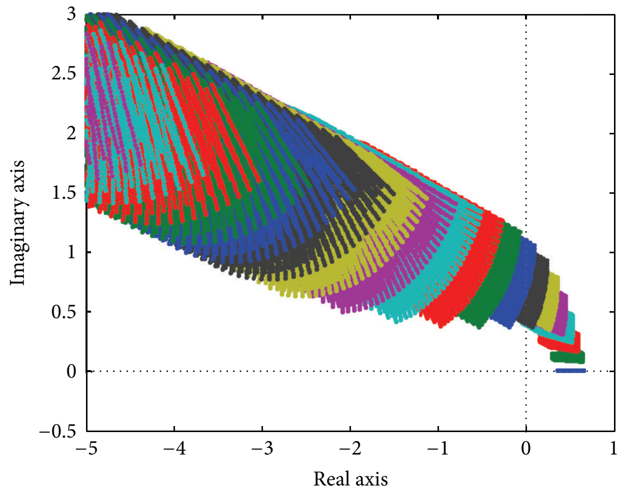

FIGURE 10: Value sets for controller (18) and plant (11) with parameters (15)—zoomed version.

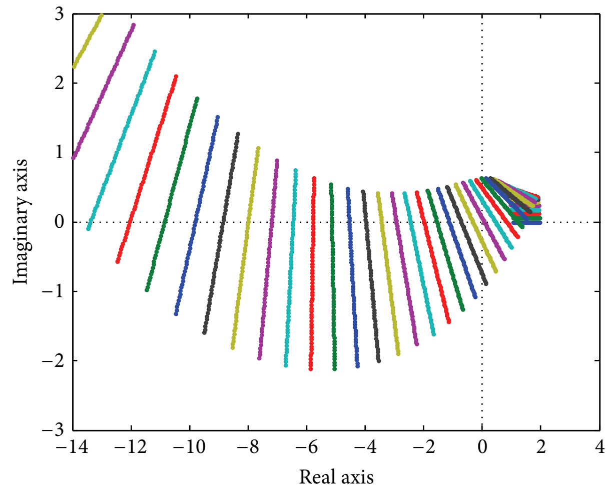

FIGURE 12: Value sets for controller (21) and plant (11) with parameters (12)—zoomed version.

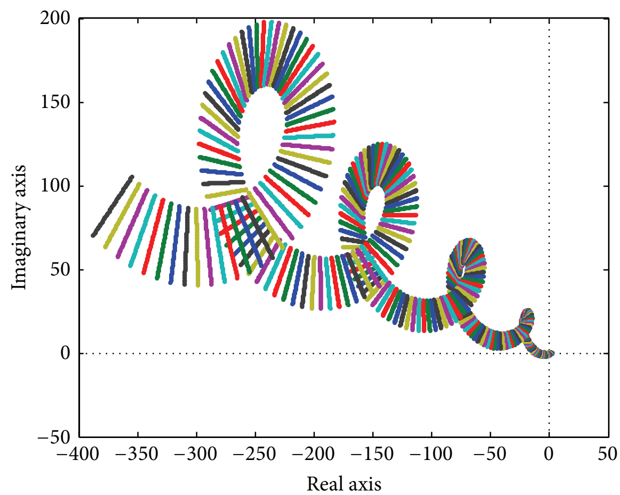

FIGURE 11: Value sets for controller (21) and plant (11) with parameters (12).

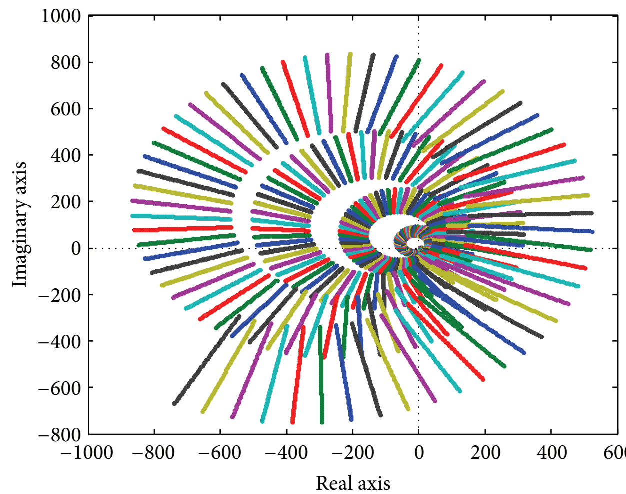

FIGURE 13: Value sets for controller (22) and plant (11) with parameters (12).

about fulfillment of all given preconditions, for example, the existence of at least one stable member of the analyzed family. If they are ignored, it can lead to the incorrect results. For example, consider again the same controlled plant with gain (12) and another PID controller with parameters:

$$C(s) = K_p + \frac{K_i}{s} + K_d s = 1.5 + \frac{0.5}{s} + 0.3s. \tag{22}$$

The corresponding value sets (again for $\omega = 0 : 0.05 : 15$) and the closer look at the origin are shown in Figures 13 and 14, respectively.

Since the zero is obviously excluded from the value sets, it could (wrongly) indicate the robust stability of the family. However, the family does not have any stable member and so the zero exclusion condition is not fulfilled actually. In fact, all members of the family are unstable which is the reason why the stability border is not crossed at all and why the zero point is not included. All in all, the family is not robustly stable and the assumed control loop would be unstable even for all possible values of K from the prescribed interval.

5. Example: Robust Stabilization of Main Irrigation Canals

Whereas the previous examples from Section 4 have demonstrated primarily the basic utilization of the method and typical shapes of the value sets, the following example is based on real control of main irrigation canal pools [13, 37].

Water is indispensable element for life and it is becoming the most valuable resource all over the world. Nowadays, irrigation is reported as the major water consuming activity [37] and thus control which will lead to more efficient water management in irrigation systems is required.

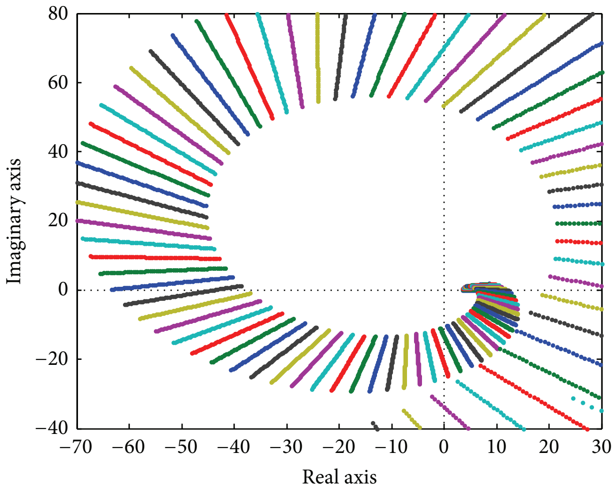

Figure 14: Value sets for controller (22) and plant (11) with parameters (12)—zoomed version.

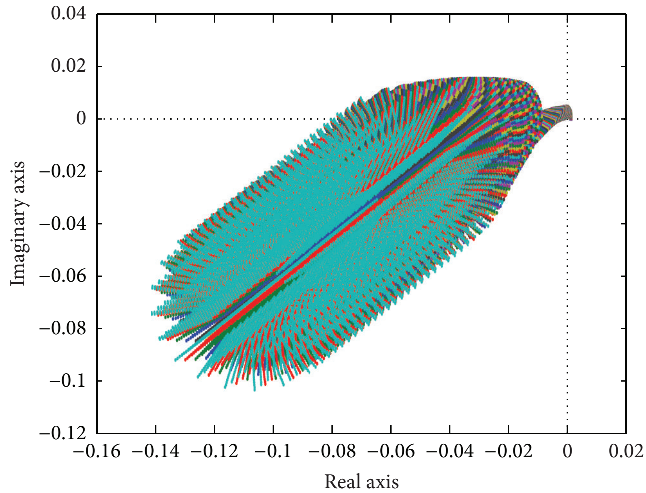

Figure 15: Value sets for controller (24) and plant (23) with parameters (26).

In [37], the main irrigation canal pool was modelled as a second-order (integer order) transfer function with time-delay:

$$G(s, K, T_1, \Theta) = \frac{K}{(T_1 s + 1)(T_2 s + 1)} e^{-\Theta s}, \tag{23}$$

where static gain K, time constant T_1, and time-delay term Θ are supposed to exhibit wide variations as a result of discharge through the upstream gate which varies in some operation range. The second time constant T_2 represents the motor and gate dynamics which is much faster than the dynamics of the canal pool and thus it is considered to be invariant. The nominal values of the uncertain parameters are $K_0 = 1.25$, $T_{10} = 300[s]$, and $\Theta_0 = 600[s]$ and the fixed constant is $T_2 = 60[s]$.

Two controllers were designed in [37]. The first one is a classical PID controller:

$$\begin{aligned} C_{\text{PID}}(s) &= K_p + \frac{K_i}{s} + K_d s \\ &= 0.5511 + \frac{0.0008}{s} + 80.1334 s \end{aligned} \tag{24}$$

and the second one has the form of fractional order PI:

$$C_{\text{FPI}}(s) = \frac{K_p s + K_i}{s^{0.66}} = \frac{1.9964 s + 0.0089}{s^{0.66}}. \tag{25}$$

The maximal assumed variations of parameters from [37], that is, $0 < K \leq 3.125$, $6 \leq T_1 \leq 6000$, and $0 < \Theta \leq 1800$, are really extreme and they lead to robustly unstable closed loop for both controllers. However, as the practical range of parameters should be much smaller, the intervals corresponding to $\pm 40\%$ of the nominal values are supposed for the sake of robust stability analysis in this paper. That is, the considered uncertain parameters in transfer function (23) are as follows:

$$\begin{aligned} K &= [0.75, 1.75]; \\ T_1 &= [180, 420]; \\ \Theta &= [360, 840]. \end{aligned} \tag{26}$$

The corresponding families of closed-loop characteristic quasipolynomials are

$$\begin{aligned} p_{\text{cl-PID}}(s, K, T_1, \Theta) &= (T_1 s + 1)(T_2 s + 1) s \\ &\quad + K e^{-\Theta s} \left(K_d s^2 + K_p s + K_i\right) \end{aligned} \tag{27}$$

for PID controller (24) and

$$\begin{aligned} p_{\text{cl-FPI}}(s, K, T_1, \Theta) &= (T_1 s + 1)(T_2 s + 1) s^{0.66} \\ &\quad + K e^{-\Theta s} \left(K_p s + K_i\right) \end{aligned} \tag{28}$$

for fractional order PI controller (25).

The sampling of frequency and parameters for the sake of the value sets visualization has been chosen as

$$\begin{aligned} \omega &= 0 : 0.0001 : 0.015 \\ K &= 0.75 : 0.05 : 1.75 \\ T_1 &= 180 : 10 : 420 \\ \Theta &= 360 : 20 : 840. \end{aligned} \tag{29}$$

The value sets for the family of quasipolynomials (27), that is, for PID controller (24) and family of systems (23) with parameters (26), are plotted in Figure 15. The zoomed complex plane origin is then shown in Figure 16. Since the family contains a stable member and the zero point is excluded from the value sets, the closed-loop control system is robustly stable.

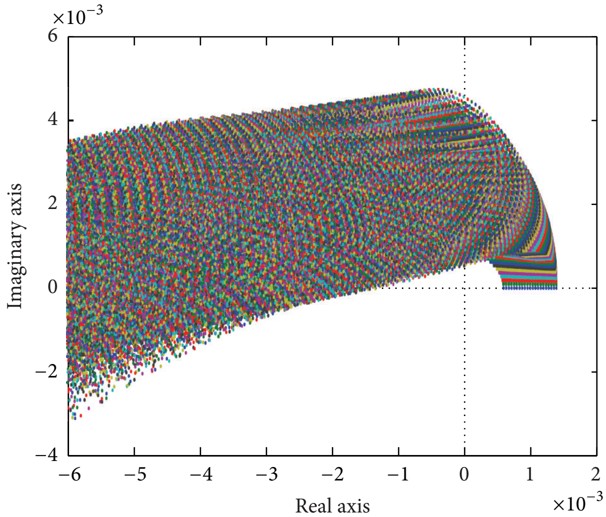

Figure 16: Value sets for controller (24) and plant (23) with parameters (26)—zoomed version.

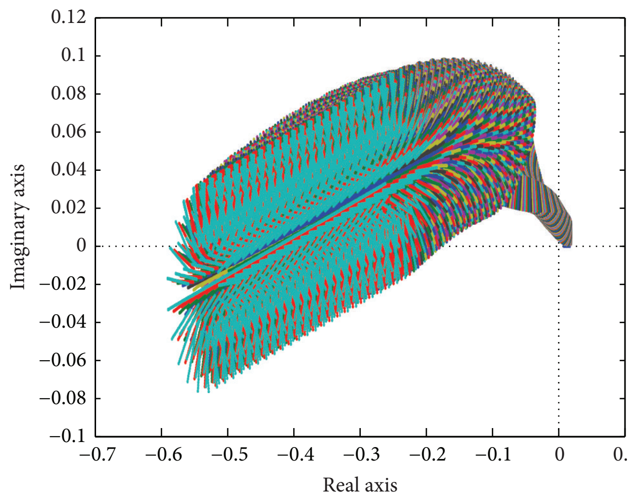

Figure 17: Value sets for controller (24) and plant (23) with parameters (26).

The value sets for the family of quasipolynomials (28), that is, for fractional order PI controller (25) and family of systems (23) with parameters (26), and the closer look are depicted in Figures 17 and 18, respectively. The final closed-loop control system is robustly stable also in this case.

6. Conclusion

The main aim of the paper was to present a graphical approach to robust stability analysis and its application to fractional order time-delay feedback control loops consisting of a family of fractional order time-delay plants and either integer order or fractional order controller. The robust stability was verified through visualization of the value sets of a closed-loop characteristic quasipolynomial family and subsequent application of the zero exclusion condition

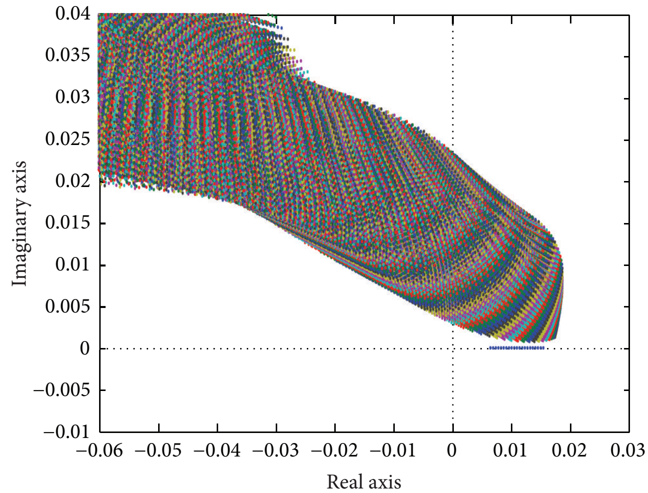

Figure 18: Value sets for controller (24) and plant (23) with parameters (26)—zoomed version.

for various combinations of uncertain parameters. Despite the fact that the presented computational examples from Section 4 combined the cases of fractional order plants with integer order controllers and the practically oriented example from Section 5 analyzed the integer order plant with integer order or fractional order controller, the combination of fractional order plant and fractional order controller is also effectively solvable by the presented graphical method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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