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Polynomial Digital Control of a Series Equal Liquid Tanks

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Abstract. Time-delays are mainly caused by the time required to transport mass, energy or information, but they can also be caused by processing time or accumulation. Typical examples of such processes are e.g. pumps, liquid storing tanks, distillation columns or some types of chemical reactors. In many cases time-delay is caused by the effect produced by the accumulation of a large number of low-order systems. Several industrial processes have the time-delay effect produced by the accumulation of a great number of low-order systems with the identical dynamic. The dynamic behavior of series these low-order systems is expressed by high-order system. One of possibilities of control of such processes is their approximation by low-order model with time-delay. The paper is focused on the design of the digital polynomial control of a set of equal liquid cylinder atmospheric tanks. The designed control algorithms are realized using the digital Smith Predictor (SP) based on polynomial approach – by minimization of the Linear Quadratic (LQ) criterion. The LQ criterion was combined with pole assignment.

1 Introduction

Time-delay may be defined as the time interval between the start of an event at one point in a system and its resulting action at another point in the system. Delays are also known as transport lags or dead times; they arise in physical, chemical, biological and economic systems, as well as in the process of measurement and computation.

Some technological processes in industrial practice are characterized by high-order dynamic behaviour or large time constants and time-delays. For control engineering, such processes can often be approximated by the FOTD (first-order-time-delay) model. Time-delay in a process increases the difficulty of controlling it. However, using the approximation of a high-order process by a low-order model with time-delay provides simplification of the control algorithms. In many cases the time delay is caused by the effect produced by the accumulation of a large number of low order systems.

Let us consider a continuous-time dynamical linear SISO system (single input $u(t)$ – single output $y(t)$) with time-delay L. The transfer function of a pure transportation lag is e^{-Ls} where s is a complex variable. Overall transfer function with time-delay is in the form

$$G_L(s) = G(s)e^{-Ls} \qquad (1)$$

where $G(s)$ is the transfer function without time-delay.

Historically first modifications of time-delay control algorithms were proposed for continuous-time (analog) controllers using various approaches. One of possible approaches to control of process with time-delay is Smith predictor [1]. In industrial practice, the implementation of the time-delay compensation algorithms on continuous-time technique is difficult. One of possible approaches to control of process with time-delay is digital Smith predictor based on polynomial theory.

This paper is oriented to design of a LQ control using polynomial theory [2, 3]. The minimization of LQ criterion is completed with pole assignment principle.

The digital pole assignment Smith predictor was designed using a polynomial approach in [4]. The design of this controller was extended by a method for a choice of a suitable pole assignment of the characteristic polynomial. Because the classical analog Smith predictor is not suitable for control of unstable and integrating time-delay processes, the polynomial digital LQ Smith predictor for control of unstable and integrating time-delay processes has been designed in [5].

It is obvious that the majority processes met in industrial practice are influenced by uncertainties. The uncertainties suppression can be solved either implementation adaptive control or robust control. Some adaptive (self-tuning) modifications of the digital Smith predictors are designed in [4, 6, 7]. Two versions of these controllers were implemented into MATLAB Toolbox [8, 9].

Main contribution of this paper is focused on the design of the digital polynomial control of a set of liquid cylinder atmospheric tanks. The paper is organized in the following way. The general problem of a control of the time-delay systems with regard to polynomial approach is described in Section 1. The high-order system (a set of *n*

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equal liquid cylinder atmospheric tanks) is analysed in Section 2. Section 3 contains description of identification procedures. The fundamental principle of digital Smith predictor is described in Section 4. The primary polynomial LQ controller, which is component of the digital Smith predictor, is proposed in Section 5. The simulation verifications of individual control-loops with their results are presented in Section 6. Section 7 concludes this paper.

2 Series of equal liquid tanks

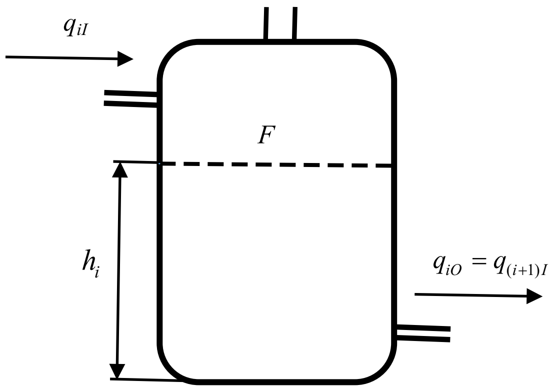

Fig. 1 Schema of liquid cylinder tank

In many cases in industrial practice the time-delay is caused by the effect produced by the accumulation of a large number of low-order systems. Consider a set of n equal cylinder atmospheric tanks, where a single tank is shown in Fig. 1 [10] and the whole set is shown in Fig. 2.

In this system, the output flow of tank i (q_{iO}) feeds tank i + 1; that is, the input flow tank i + 1 is $q_{(i+1)I} = q_{iO}$. If all the tanks have the same area (F) of crosscut and the individual tank levels are near to an operating point, then the dynamic behaviour of the level in each tank h_i can be modelled by a linear system

$$F\frac{dh_i}{dt} = q_{iI} - q_{io} \qquad q_{iO} = K_1 h_i \tag{2}$$

where K_1 is a constant that depends on the tank characteristics and $T = F / K_1$ is time constant.

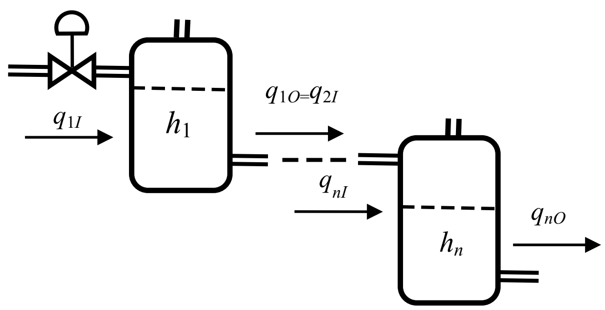

Fig. 2 Series of liquid cylinder tanks

Consider a set of n tanks as shown in Fig. 2. Thus, the transfer function relating the input follow in tank i and its level is given by

$$h_i(s) = \frac{1/K_1}{Ts+1} q_{iI}(s) \tag{3}$$

For tank 1 is

$$h_1(s) = \frac{1/K_1}{Ts+1} q_{1I}(s) \tag{4}$$

and for tank 2 using the second equation of (2)

$$h_2(s) = \frac{1/K_1}{Ts+1} q_{2I}(s) = \frac{1/K_1}{Ts+1} q_{1O}(s) = \frac{1/K_1}{Ts+1} K_1 h_1(s) \tag{5}$$

Then, using the expression (4) it follows

$$h_n(s) = G(s) q_{1I}(s) = \frac{K_g}{(Ts+1)^n} q_{1I}(s) \tag{6}$$

and the transfer function of the series of tanks system is

$$G(s) = \frac{h_n(s)}{q_{1I}(s)} = \frac{K_g}{(Ts+1)^n} \tag{7}$$

where $K_g = 1/K_1$ is static gain of the system (7).

Consider for simulation experiments of control model (7) the eight – order system, i. e. n = 8. Following parameters of the individual liquid tanks are considered (see Fig. 1):
high of tank h = 1.5 m;
diameter of tank d_T = 1 m;
tank area $F = \frac{\pi d_T}{4} = 0.785$ m^2;
time constant T = 2 min;
constant $K_1 = \frac{F}{T} = \frac{0.785}{2} = 0.3925$ m^{2}min^{-1};
static gain $K_g = \frac{1}{K_1} = \frac{1}{0.3925} = 3.08$ m^{-2} min.

The resulting transfer function is given by

$$G(s) = \frac{h_8(s)}{q_{1I}(s)} = \frac{3.08}{(2s+1)^8} \tag{8}$$

If (8) is the transfer function of a continuous-time dynamic system, then the following expression for the discrete transfer function with zero - order holder and sampling period T_0 is valid

$$G(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2} + \ldots + b_8 z^{-8}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_8 z^{-8}} \tag{9}$$

The transfer function (9) was approximated by the discrete second-order model with time-delay

$$G_L(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} z^{-d} = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} z^{-d} \tag{10}$$

3 Identification of series liquid tanks

3.1 Determination of number time-delay steps

The number of time-delay steps is obtained using an off-line identification by the least squares method (LSM). The measured process output (liquid level $h_8(k)[\mathrm{m}]$ near operating flow) is influenced by input – generator of white noise which excites changes of flow rate $q_{1I}(k)\left[\mathrm{m}^3\,\mathrm{min}^{-1}\right]$. The non-measurable system disturbances cause errors **e** in the determination of model parameters and therefore real output vector is in the form

$$\mathbf{y} = \mathbf{F}\mathbf{\Theta} + \mathbf{e} \tag{11}$$

The matrix **F** has dimension (*N*-*n*-*d*, 2*n*), the vector **y** (*N*-*n*-*d*) and the vector of parameter model estimates $\hat{\mathbf{\Theta}}$ (2*n*). *N* is the number of samples of measured input and output data, *n* is the model order. It is possible to obtain the LSM expression for calculation of the vector of the parameter estimates

$$\hat{\mathbf{\Theta}} = \left(\mathbf{F}^{\mathrm{T}}\mathbf{F}\right)^{-1}\mathbf{F}^{\mathrm{T}}\mathbf{y} \tag{12}$$

Equation (12), where n = 8, serves for calculation of the vector of the parameter estimates $\hat{\mathbf{\Theta}}$ using *N* samples of measured input-output data. The form of individual vectors and matrices in equations (11) and (12) are introduced in [11].

Consider that model (10) is the deterministic part of the stochastic process described by the ARX (regression) model

$$\begin{aligned} y(k) &= -a_1 y(k-1) - a_2 y(k-2) + \\ &+ b_1 y(k-1-d) + b_2 y(k-2-d) + e_s(k) \end{aligned} \tag{13}$$

where $e_s(k)$ is the random non-measurable component.

The vector of parameter model estimates is computed by solving equation (12)

$$\hat{\mathbf{\Theta}}^T(k) = \begin{bmatrix} \hat{a}_1 & \hat{a}_2 & \hat{b}_1 & \hat{b}_2 \end{bmatrix} \tag{14}$$

and is used for computation of the predicted output

$$\begin{aligned} \hat{y}(k) &= -\hat{a}_1 y(k-1) - \hat{a}_2 y(k-2) + \\ &\hat{b}_1 u(k-1-d) + \hat{b}_2 u(k-2-d) \end{aligned} \tag{15}$$

The quality of identification can be considered according to error, i.e. the deviation

$$\hat{e}(k) = y(k) - \hat{y}(k) \tag{16}$$

Continuous-time system (8) was identified by discrete model (10) using off-line LSM (12) for different time-delay dT_0; $T_0 = 1\,\mathrm{min}$. The White Noise Generator was used as excitation input signal. A criterion of the identification quality is based on sum of squares of error

$$J_{\hat{e}^2}(d) = \sum_{k=1}^{N} \hat{e}^2(k) \tag{17}$$

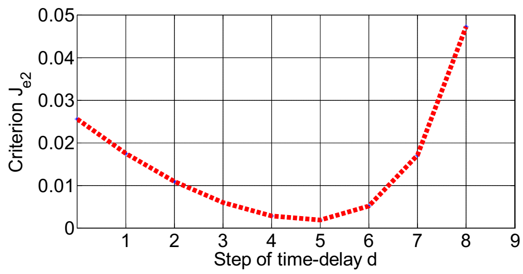

Fig. 3 Criterion of Quality Identification for $d \in [0, 8]$

This criterion represents accuracy of process identification. It is obvious from Fig. 4 that minimum value of the criterion (21) is reached when the number of time-delay steps d = 5. Then it is possible to use model

$$\hat{G}_L\left(z^{-1}\right) = \frac{\hat{b}_1 z^{-1} + \hat{b}_2 z^{-2}}{1 + \hat{a}_1 z^{-1} + \hat{a}_2 z^{-2}} z^{-5} \tag{18}$$

for an approximation of model (8).

3.2 Identification procedure

The model (18) was used in off-line identification procedure for calculation of parameter estimates. This procedure was realized by the MATLAB function

$$x = \mathrm{fminsearch}('\mathit{name\_fce}', x_0) \tag{19}$$

from the MATLAB Optimization Toolbox. This function finds minimum of an unconstrained multivariable function using derivative-free method. Algorithm "fminsearch" uses the simplex search method of [12]. This is a direct search method that does not use numerical or analytic gradients.

The difference between static gain K_g = 3.08 of the continuous-time transfer function (8) and estimation of the static gain of discrete transfer function (18)

$$\hat{K}_g = \frac{\hat{b}_1 + \hat{b}_2}{1 + \hat{a}_1 + \hat{a}_2} \tag{20}$$

can serve as a good criterion for the quality of identification.

Results of experimental identification demonstrated that better the identification quality is obtained using "fminsearch" method. Discrete model for sampling period $T_0 = 1\,\mathrm{min}$

$$\hat{G}_L\left(z^{-1}\right) = \frac{0.0309z^{-1} + 0.0286z^{-2}}{1 - 1.777z^{-1} + 0.7964z^{-2}} z^{-5} \tag{21}$$

is obtained using this method with static gain (20) K_g = 3.08 is the same as in model (8). Comparison of step responses of continuous-time (8) and discrete model (21) is shown in Fig. 4. The input step signal $\Delta q_{1I} = 0.04\,\mathrm{m}^3\,\mathrm{min}^{-1}$ was chosen so that tank level is near to an operating point.

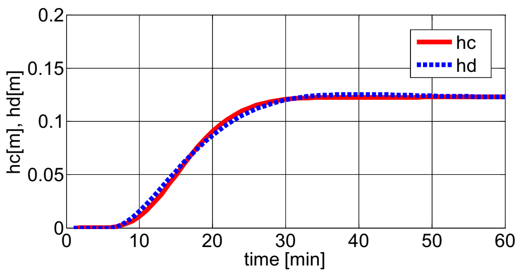

Fig. 4 Comparison of step responses of models (8) and (20)

It is obvious that linear model (20) was obtained without complying with valves contain hysteresis and other nonlinearities that the series liquid tanks system contains [13].

4 Fundamental principle of digital SP

The block diagram of a digital SP (see [4], [5]) is shown in Fig. 5. The function of the digital version is similar to the classical analogue version. In this paper, the second-order linear model (20) with the time-delay $d = 5$ is considered. The block $G_m(z^{-1})$ represents process dynamics without the time-delay and is used to compute an open-loop prediction. The numerator in transfer function $G_d\left(z^{-1}\right)$ is replaced by its static gain $B(1)$, i.e. for $z = 1$. This is to avoid problem of controlling a model with a $B(z^{-1})$, which has non-minimum phase zeros caused by a high sampling period or fractional delay. Since $B(z^{-1})$ is not controllable as in the case of a time-delay, it is moved out of the prediction model $G_m(z^{-1})$ and is treated together with the time-delay block, as shown in Fig. 5.

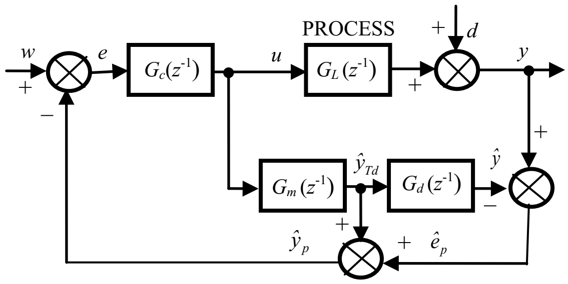

Fig. 5 Block diagram of a digital Smith predictor

The difference between the output of the process y and the model including time-delay $\hat{y}$ is the predicted error $\hat{e}_p$ as shown in Fig. 1, whereas e and d are the error and the measured disturbance, w is the reference signal. The primary (main) controller $G_c(z^{-1})$ can be designed by different approaches (for example digital PID control or methods based on polynomial approach). The detailed description of the block diagram (Fig. 5) is in [2].

5 Design of primary 2DOF controller

The design of the control algorithm is based on a general block scheme of a closed-loop with two degrees of freedom (2DOF) according to Fig. 6. The controller synthesis consists in the solving linear polynomial (Diophantine) equations [14]. From first polynomial equation

$$A(z^{-1})K(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1}) = D(z^{-1}) \quad (22)$$

it is possible to compute 7 feedback controller parameters – coefficients of the polynomials Q, P. Polynomial $D\left(z^{-1}\right)$ is the characteristic polynomial and $K\left(z^{-1}\right) = 1 - z^{-1}$.

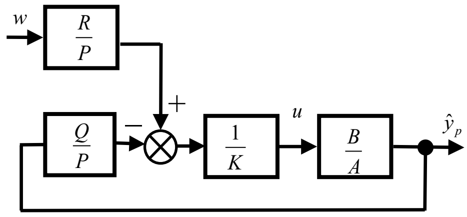

Fig. 6 Block diagram of a closed loop 2DOF control system

Asymptotic tracking of the reference signal w is provided by the feedforward part of the controller which is given by solution of the second following polynomial Diophantine equation

$$S(z^{-1})D_w\left(z^{-1}\right) + B(z^{-1})R(z^{-1}) = D(z^{-1}) \quad (23)$$

For a step-changing reference signal value, polynomial $D_w(z^{-1}) = 1 - z^{-1}$ and S is an auxiliary polynomial which does not enter into the controller design. Then it is possible to derive the polynomial R from equation (24) by substituting $z = 1$

$$R = r_0 = \frac{D(1)}{B(1)} \quad (24)$$

The 2DOF controller output is given by

$$u(k) = \frac{r_0}{K\left(z^{-1}\right)P\left(z^{-1}\right)}w(k) - \frac{Q\left(z^{-1}\right)}{K\left(z^{-1}\right)P\left(z^{-1}\right)}\hat{y}_p(k) \quad (25)$$

The LQ control methods try to minimize the quadratic criterion which uses penalization of the value of the controller output

$$J = \sum_{k=0}^{\infty}\left\{\left[w(k) - \hat{y}_p(k)\right]^2 + q_u\left[u(k)\right]^2\right\} \quad (26)$$

where q_u is the so-called penalization constant, which gives the influence of the controller output to the value of the criterion. In this paper, criterion minimization (26) will be realized through the spectral factorization for an input-output description of the system

$$A(z)q_uA\left(z^{-1}\right) + B(z)B\left(z^{-1}\right) = D(z)\delta D\left(z^{-1}\right) \quad (27)$$

where δ is a constant chosen so that $d_0 = 1$. $A(z)$, $B(z)$ are the second-order polynomials and $D(z)$ is also the second-order polynomial

$$D\left(z^{-1}\right) = 1 + d_1z^{-1} + d_2z^{-2} \quad (28)$$

The design of the LQ controllers for control of the second-order system with time-delay (10) is in detail derived in [4, 15]. It is obvious from (28) that using spectral factorization of the second-order model, only two parameters d_1 and d_2 can be computed. Simulation experiments demonstrated that use of the second-order polynomial (28) leads to the oscillations of the controller output for control of the time-delay system (21). These oscillations it is impossible to eliminate not even by increasing of penalization factor q_u. This problem it is possible to solve by addition another pole into polynomial (28). It means to use the third order polynomial

$$D_3\left(z^{-1}\right) = 1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3} \qquad (29)$$

for controller design. The polynomial $D_3(z^{-1})$ is then to use in equations (22) and (23). The polynomial (29) may have three different real poles α, β, γ or one complex conjugated pole $z_{1,2} = \alpha \pm j\beta$ and one real pole γ. These poles must be included into polynomial $D_3(z^{-1})$ (29). A suitable pole assignment was designed for both types of the processes:

1st possibility:
Polynomial (29) has two different real poles α, β (computed from (27)) and user-defined real pole γ. Then it is possible to write polynomial (29) as a product root of factor

$$D_3(z) = (z-\alpha)(z-\beta)(z-\gamma) \qquad (30)$$

and its individual parameters can be expressed as

$$d_1 = -(\alpha+\beta+\gamma); \quad d_2 = \alpha\beta + \alpha\gamma + \beta\gamma; \quad d_3 = \alpha\beta\gamma \ (31)$$

2nd possibility:
Polynomial (29) has the complex conjugate pole $z_{1,2} = \alpha \pm j\beta$ (computed from (27)) and user-defined real pole γ. Then polynomial (30) has the form

$$D_3(z) = (z-\alpha-j\beta)(z-\alpha+j\beta)(z-\gamma) \qquad (32)$$

and its individual parameters can be expressed as

$$d_1 = -(2\alpha+\gamma); \quad d_2 = 2\alpha\gamma + \alpha^2 + \beta^2; \quad d_3 = -\left(\alpha^2+\beta^2\right)\gamma \qquad (33)$$

With increased penalization constant $q_u \geq 0$, the amplitude of the controller output decreases and thereby, the flow of the process output is smoothened and any possible oscillations or instability are damped.

6 Simulation verification and results

A simulation verification of the designed control algorithm was performed in MATLAB/SIMULINK environment. It is obvious from Section 5 that courses of the controlled variables are dependent not only on pole assignment obtained by spectral factorization (α, β), but also on the user-defined real γ and the penalization factor q_u.

6.1 Influence of pole γ on quality of control

The following individual simulation experiments were realized subsequently: the penalization factor was set as q_u = 10 and the user-defined real pole γ was increasing. The courses of the controlled process output and controller output for individual poles γ are shown in Fig. 7 – 9.

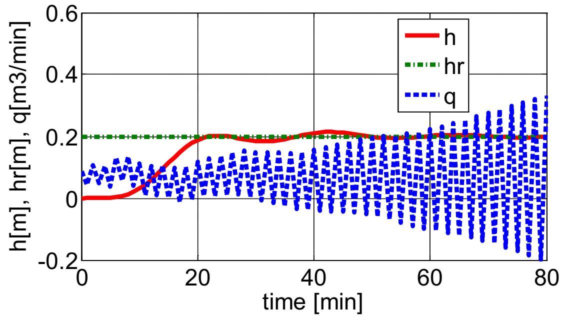

Fig. 7 Control courses for parameters $q_u = 10; \gamma = 0.01$

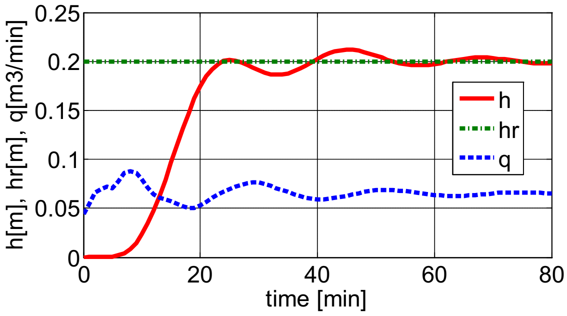

Fig. 8 Control courses for parameters $q_u = 10; \gamma = 0.5$

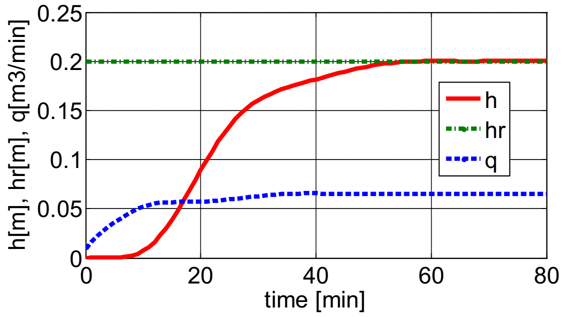

Fig. 9 Control courses for parameters $q_u = 10; \gamma = 0.9$

It is obvious from Fig. 7 - 9 that increasing the value of the pole γ caused a slowdown of the process output y and suppression of oscillations. Oscillations of the controller output were also suppressed.

6.2 Influence of penalization factor q_u on quality of control

The following simulation experiments were realized subsequently: the real pole was set as γ = 0.5 and the penalization factor q_u was increasing. The courses of the controlled process output and controller output for individual poles γ are shown in Fig. 10 – 12.

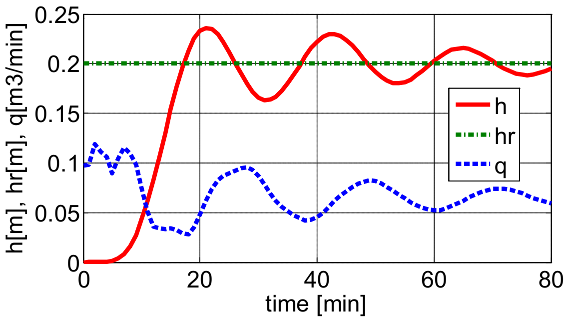

Fig. 10 Control courses for parameters $\gamma = 0.5; q_u = 1$

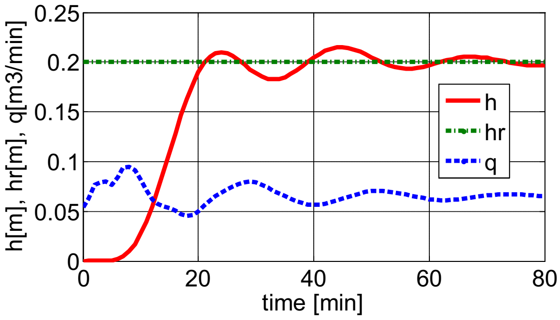

Fig. 11 Control courses for parameters $\gamma = 0.5; q_u = 5$

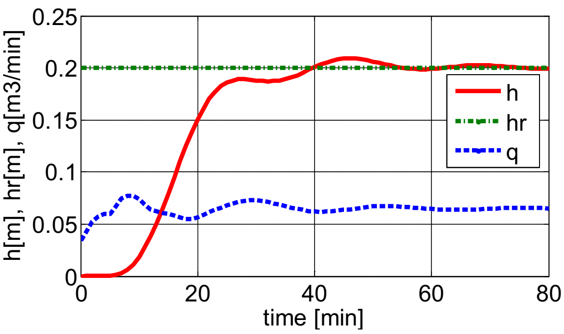

Fig. 12 Control courses for parameters $\gamma = 0.5; q_u = 100$

It is obvious from Fig. 10 - 12 that increasing the value of the penalization factor q_u caused a slowdown of both the process output y and the manipulated variable u and also suppression of oscillations.

7 Conclusion

The paper presents a simulation control of the high-order process (a set of equal liquid tanks). This process was identified by the second-order model with five time- delay steps. The designed control algorithm uses the digital LQ Smith predictor. A minimization of the LQ criterion was combined with the pole assignment principle. The designed controller was verified by simulation in the MATLAB/SIMULINK environment. The results of simulation verifications demonstrated very good control quality and robustness of the designed digital LQ algorithm. Another good property of the designed algorithm is the possibility of suitable penalization of the manipulated variable which prevents oscillations of the actuator (in this case the input valve). The main contribution of this paper is the finding that a high-order system, which is composed of a set of low-order systems, can be approximated by a low-order model with time-delay. For this approximated model it is possible to design relatively simple digital controllers. The polynomial controller was derived purposely by the analytical way (without utilization of numerical methods) in order to obtain an algorithm with easy feasibility in industrial practice.

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