The 18th INTERNATIONAL DAAAM SYMPOSIUM
"Intelligent Manufacturing & Automation: Focus on Creativity, Responsibility and Ethics of Engineers"
24-27th October 2007

OPTIMIZATION OF HEAT SOURCE WITH COMBINED PRODUCTION OF HEAT AND ELECTRIC ENERGY

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Abstract: *This paper deals with the continuation and improvement of non-linear model which was published in paper (Balátě et al., 2006b). The paper describes access to solving optimisation of operation of heat source with combined production of heat and power by methods of non-linear programming.*

Key words: *heat, power, CHP – combined production of heat and power, non-linear optimization*

1. INTRODUCTION

The problem of economical distribution of load between cooperating production units arises at optimization of combined production of electric energy and heat production plants which have larger number of cooperating production units. The basic presumption of economical production is knowledge of power-economical characteristics of separate production appliances, in our specific case it concerns consumption characteristics of boilers which generally have non-linear course.

The task of economical distribution of load belongs between the basic tasks of optimal control. In this case its basic principle is to minimize fuel consumption for required heat output. From the mathematical point of view the aim of optimal control will be to achieve extreme value of objective (criterial) function E, in our case minimization of production costs. We will look for the minimum of objective function E, therefore for the minimum of production costs.

In our case the optimization of combined heat and power plant operation can be carried out by two methods:

- linear replacement of consumption characteristics of boilers,
- non-linear replacement of consumption characteristics of boilers.

In following chapters are described linear as well as non-linear mathematical models of a heat source with combined production of heat and electric energy.

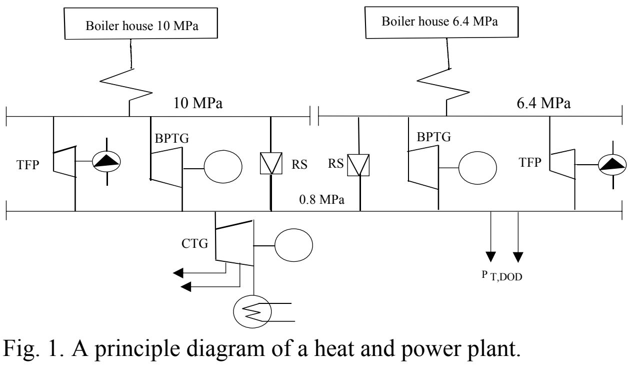

Fig. 1. A principle diagram of a heat and power plant.

Legend: TFP – Turbo-Feed-Pump, BPTG – Back-Pressure-Turbo-Generator, CTG – Condensing-Turbo-Generator, RS – Reducing Station, $P_{T,DOD}$ – Heat output supply

The total immediate electric output *P* will be calculated according to determined heat output supplied to the heat network $P_{T,DOD}$. $P_{T,DOD}$ is the independent variable. There are two methods as the solutions to form mathematical models:

- linear,
- non-linear model.

2. MATHEMATICAL LINEAR MODEL

This model was formulated in these papers (Balátě, 1969), (Balátě et al., 2006a), (Balátě et al., 2006b), (Phan, 1996), (Šimek, 2007). Essentially the linear model was created on the base of knowledge of balance equations of steam pipings, further of limiting conditions of non-negative values of dependent variables and of objective function. All variables in the used mathematical model and also limiting conditions are linear. Independent variable is supplied heat output in the heat network $P_{T,DOD}$. In order to form the mathematical linear model, it is necessary to perform the following requirements:

- the consumption parameters of production units must have a convex monotonic continuance,
- it is necessary to approximate the convex curves of the consumption parameters by linear sections.

The way of linearization of the consumption parameters characteristics is illustrated on the Fig. 2a.

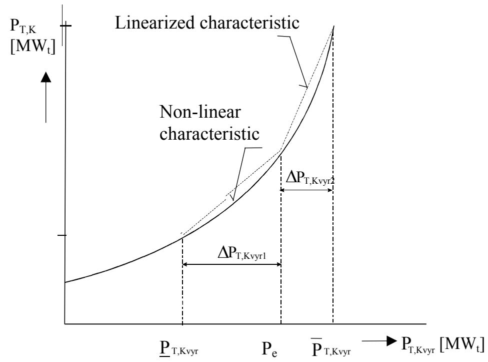

Fig. 2a. Economic characteristic of steam boiler and its linearization.

Consumption characteristics for back-pressure turbines are shown on Fig. 2b.

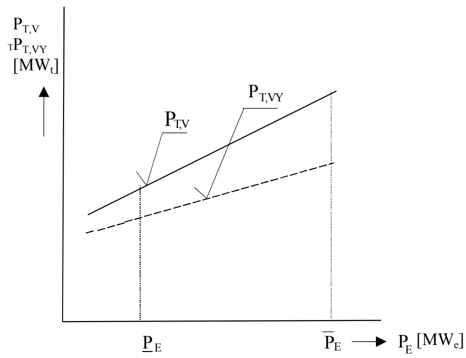

Fig. 2b. Economic characteristic of back pressure turbine.

The linear mathematical model for linear programming consists:
- heat balance equations in piping,
- equations of produced electric power,
- definition of non-negative inequalities,
- definition of objective function E,
- determination of delivered heat output to heat network.

The linear model has a lot of dependent variables and one independent variables – $P_{T,DOD}$

3. MATHEMATICAL NON-LINEAR MODEL

In this chapter we will bring near creation of non-linear mathematical model. All denominations used for linear mathematical models are used also for non-linear mathematical model.

Derivative of consumption characteristics:

Consumption characteristics of boilers have generally non-linear courses (Fig. 2a). These courses can be replaced by exponential approximation:

$$y = a.e^{\beta.x} \quad (1)$$

their relative increment of heat is expressed by derivative

$$y' = a.\beta.e^{\beta.x} \quad (2)$$

Non-linear mathematical model of the scheme of combined heat and electric energy production plant (Fig. 1) is described by the following equations:

1. Balance equation for steam piping 10 MPa is valid:

$$\sum_{i=1}^{n,10} a_{10,i} e^{\beta_{10,i} P_{T,Kvyr}^{10,i}} = \left[\left(\sum_{j=1}^{m} P_{T,T,V}^{10,j} \right) + P_{T,N,V}^{10} + P_{T,R}^{10/0.8} \right] k_q^{10} \quad (3)$$

Left side in this balance equations is expressed by means of exponential approximation (1); separate coefficients are expressed as follows: $a = a_{10,i}$, $\beta = \beta_{10,i}$ and dependent variable which we look for is expressed as: $x = P_{T,Kvyr}^{10,i}$.

2. Balance equation for steam piping 6.4 MPa is valid:

$$\sum_{i=1}^{n,6.4} a_{6.4,i} e^{\beta_{6.4,i} P_{T,Kvyr}^{6.4,i}} = \left[\left(\sum_{j=1}^{m} P_{T,T,V}^{6.4,j} \right) + P_{T,N,V}^{6.4} + P_{T,R}^{6.4/0.8} \right] k_q^{6.4} \quad (4)$$

Left side of this balance equation is also expressed by means of exponential approximation (1); separates coefficients are expressed $a = a_{6.4,i}$, $\beta = \beta_{6.4,i}$ and dependent variable which we look for is expressed as: $x = P_{T,Kvyr}^{6.4,i}$.

3. Balance equation for steam piping 0.8 MPa is valid:

$$\sum_{j=1}^{m} P_{T,T,VY}^{10,j} + \sum_{j=1}^{m} P_{T,T,VY}^{6.4,j} + P_{T,N,VY}^{10} + P_{T,N,VY}^{6.4} + P_{T,R}^{10/0.8} + P_{T,R}^{6.4/0.8} = \left[\left(\sum_{j=1}^{m} P_{T,T,V}^{0.8,j} \right) + P_{T,el} + P_{DOD} \right] k_q^{0.8} \quad (5)$$

4. Total produced electric output is therefore:

$$P_E = \sum_{j=1}^{m,10} b_{T,V}^{10,j} P^{10,j} + \sum_{j=1}^{m,6.4} b_{T,V}^{6.4,j} P^{6.4,j} + \sum_{j=1}^{m,0.8} P^{0.8,j} \quad (6)$$

5. Objective function:

Increments of output produced by boilers $\Delta P_{T,Kvyr,s}$ (dependent variables) multiplied by pertinent relative increments of heat consumption of boilers $b_{k,s}$ (cost coefficients) (Balátě, et al., 2006b) and their sum for boilers operating in steam pipings of operational pressure 10 MPa and 6.4 MPa are summarized in the objective function E (7). Objective function expresses production costs for dependent variable $P_{T,Kvyr.}$ It is necessary to calculate the values of these dependent variables for each value of independent variable $P_{T,DOD}$ – heat supplied to heat network (Fig. 1).

Then the objective function E has this form:

$$E = \sum_{i=1}^{n,10} a_{10,i} \left[e^{\beta_{10,i} P_{T,Kvyr}^{10,i}} - e^{\beta_{10,i} \underline{P}_{T,Kvyr}^{10,i}} \right] + \sum_{i=1}^{n,6.4} a_{6.4,i} \left[e^{\beta_{6.4,i} P_{T,Kvyr}^{6.4,i}} - e^{\beta_{6.4,i} \underline{P}_{T,Kvyr}^{6.4,i}} \right] \quad (7)$$

upon the conditions:

$$\underline{P}_{T,Kvyr}^{10,i} \le P_{T,Kvyr}^{10,i} \le \overline{P}_{T,Kvyr}^{10,i} \quad (i = 1, 2, \ldots, n_{10}) \quad (8)$$

$$\underline{P}_{T,Kvyr}^{6.4,i} \le P_{T,Kvyr}^{6.4,i} \le \overline{P}_{T,Kvyr}^{6.4,i} \quad (i = 1, 2, \ldots, n_{6.4}) \quad (9)$$

It is useful to stress that non-linearities of this problem are in difference of these (following) variables:

$e^{\beta_{10,i} P_{T,Kvyr}^{10,i}} - e^{\beta_{10,i} \underline{P}_{T,Kvyr}^{10,i}}$ and $e^{\beta_{6.4,i} P_{T,Kvyr}^{6.4,i}} - e^{\beta_{6.4,i} \underline{P}_{T,Kvyr}^{6.4,i}}$

Variables $e^{\beta_{10,i} \underline{P}_{T,Kvyr}^{10,i}}$ and $e^{\beta_{6.4,i} \underline{P}_{T,Kvyr}^{6.4,i}}$ are constant.

Non-linear mathematical model is described by equations No. (3)-(9).

4. CONCLUSION

This paper has explained two methods to form mathematical model of a heat and power plant which produce combination of heat and electric energy in order to optimisation its operation.

Certainly, the difference between both models consist in reduction of the number of base (dependent) variables, the consumption characteristic approximation of non-linear equations is more accurately and forming of the other way of objective function.

In the case of the linear model, the objective function is linear (the consumption proportionate increments of boiler b_K are constant) and it is non-linear in the second one (the consumption proportionate increments of boiler b_K depend on index function). In order we would gain the values for the general coefficients in the above mentioned mathematical models, it is necessary to process the whole rank of bases and consumption parameters of the production units. The obtained results would be used as background for operative planning and own operation control of production of heat and power energy in real time too.

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Acknowledgement: This work was supported in part by the Grant agency of Czech Republic under grant No: 101/06/0920 and in part by the Ministry of Education of the Czech Republic under grant No. MSM 7088352102.