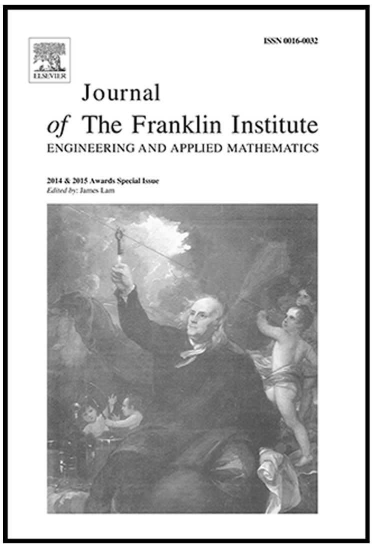
Journal Pre-proof



Fractional Order Proportional Derivative Control for Time Delay Plant of the Second Order: The Frequency Frame

Bilal Şenol , Uğur Demiroğlu , Radek Matušů

PII: S0016-0032(20)30439-7
DOI: https://doi.org/10.1016/j.jfranklin.2020.06.016
Reference: FI 4649

To appear in: *Journal of the Franklin Institute*

Received date: 25 December 2019
Revised date: 1 May 2020
Accepted date: 17 June 2020

Please cite this article as: Bilal Şenol , Uğur Demiroğlu , Radek Matušů , Fractional Order Proportional Derivative Control for Time Delay Plant of the Second Order: The Frequency Frame, *Journal of the Franklin Institute* (2020), doi: https://doi.org/10.1016/j.jfranklin.2020.06.016

Fractional Order Proportional Derivative Control for Time Delay Plant of the Second Order: The Frequency Frame

Bilal Şenol[1]*, Uğur Demiroğlu[2], Radek Matušů[3]

[1]*Department of Computer Engineering, Inonu University, Malatya, Turkey*

[2]*Department of Computer Technologies, Firat University, Elazığ, Turkey*

[3]*Centre for Security, Information and Advanced Technologies (CEBIA-Tech), Faculty of Applied Informatics, Tomas Bata University in Zlín, Zlín, Czech Republic*

**bilal.senol@inonu.edu.tr*

Abstract: This paper intends to tune fractional order proportional derivative controller for the performance, stability and robustness of second order plus time delay plant. The tuning method is based on the previously proposed “frequency frame” which is a rectangular frame enclosing gain and phase margins limited with gain and phase crossover frequencies in the Bode plot. Edges of the frame are tuned to achieve desired crossover frequencies and margins. By shaping the curves of the Bode plot, improvements are observed in the performance and robustness of the second order plus time delay system controlled by a fractional order proportional derivative controller. Generalized equations to obtain the parameters of the fractional order proportional derivative controller for second order plus time delay plant are given. In contrast to existing studies, this method reduces mathematical complexity when providing desired properties. Three examples are considered and effectiveness of the frequency frame is shown.

Keywords – Frequency Frame, phase, gain, crossover frequency, margin, FOPD, SOPTD, Bode.

1. Introduction

Fractional order calculus (FOC) had an explicit impact on many processes since the first idea came up in the late 17th century [1-3]. A fractional order differential equation can have orders of real numbers. Due to its superior performance comparing to the classical case, such motivation [4] is implemented on physics [5], electrical circuits [6] and fractances [7], signal processing [8], mechatronic systems [9], robotics [10] and neural networks [11-13] etc. This performance is achieved by the fine tuning capability brought by non-integer order differentiation thus, better system response can be achieved by this new perspective. Lately, number of the researches on control theory also had a significant increase [14-17].

Proportional integral derivative (PID), as being the leading controllers in so many industrial processes, are modified in a fractional order way of thinking as $PI^{\lambda}D^{\mu}$ controllers [18, 19]. Besides, well-known proportional integral (PI) and proportional derivative (PD) controllers are also reconsidered as PI^{λ} and PD^{μ} respectively. This paper focuses on fractional order proportional derivative (FOPD) controllers. The last decades brought a large number of studies related to FOPD controllers. For example, Li et al. proposed a FOPD controller tuning algorithm in [20] and tuning rules of a FOPD motion controller in [21]. Tuning method of FOPD controllers for path tracking of tractors is presented in [22]. General robustness analysis of FOPD controllers and time-constant robust analysis of FO[PD] controllers are studied in [23, 24]. FOPD controllers also have applications on real-world processes. FOPD and FO[PD] controllers are researched on hydraulic servo system in [25] and DC motor control is utilized by a FOPD controller in [26]. Achievable performance region computation for a FOPD motion controller is studied in [27]. The list of FOPD related studies can be widely extended.

This paper aims to tune FOPD controllers for time delay plants in the second order (SOPTD) considering four frequency properties of magnitude and phase. SOPTD plants are recently used on describing a numerous number of industrial processes such as heat exchangers [28]. There proposed a considerable amount of studies related to controller tuning for SOPTD plants. For instance, tuning of PID controllers for critically damped SOPTD systems is studied in [29]. A valuable study on modeling of stable and unstable SOPTD systems can be found in [30]. A review on PID tuning rules for SOPTD inverse response processes is presented in [31] and similar to the study in this paper, FOPD controller design for second order systems with pure time delay is given in [32].

The method implemented in this paper, namely the frequency frame shows a new perspective when comparing to existing studies in the similar direction. Most of the studies which focus on tuning controller parameters consider only one frequency specification which is usually the gain crossover frequency. For example, Wang et al. studied on non-integer order proportional integral (FOPI) tuning for time delay plants in the first order (FOPTD) in [33]. The method in the reference is based on tuning the controller by considering two specifications which are the gain crossover frequency and the phase margin. Similarly, Luo and Chen proposed the FO[PD] controller for robustness [34]. Likewise, the method tunes the gain crossover frequency for its purpose. In spite of above studies, the frequency frame considers both gain and phase crossover frequencies to compute the parameters of the FOPD controller.

Main idea of this paper is to shape the phase curve between gain and phase crossover frequencies. A rectangular frame is drawn limited by gain and phase crossover frequencies from left and right. Lengths of edges of the frame are tuned to obtain these frequency values towards researcher's desire. As known, flattening the phase curve provides improved robustness to the system to resist gain changes. This flattening procedure is used in some existing studies [35-38]. However, in these studies, the phase is flattened by setting the phase derivative to zero at a desired frequency value. In spite of the existing studies, this paper provides flatness by tuning the edges of the frequency frame. Thus, mathematical complexity is significantly reduced.

The frequency frame was firstly proposed for tuning FOPI controllers for FOPTD plants in [39]. Distinctly from the first study, this paper implements the approach on tuning a FOPD controller for a time delayed plant of the second order. Main motivation of this study is to improve the performance and robustness of the SOPTD plant with a fractional order controller which lacks the integral operator. This method aims to tune the controller parameters to satisfy both desired gain and phase crossover frequencies as mentioned in the previous paragraphs. Besides, the frequency frame approach considerably reduces the mathematical complexity with its different perspective. A fractional order controller is preferred in this paper since, satisfying both gain and phase crossover frequency values requires two different controllers. The first controller is tuned to satisfy desired phase margin at the desired gain crossover frequency. Then, the second one is tuned to provide desired gain margin at the desired phase crossover frequency. To combine these two controllers, there is the necessity of a common variable which is the fractional order μ of the controller. Thus, one controller will be obtained satisfying frequency specifications towards researcher's desire at the same time. Another purpose is to deal with the compelling structure of the FOPD controller.

This paper utilizes the frequency frame approach for tuning FOPD controllers for SOPTD plants. The method gives generalized equations to compute all unknown parameters of the FOPD controller. Four specifications of gain crossover frequency, phase crossover frequency, phase margin and gain margin are considered. As known, working with PD or FOPD controllers mostly yield the system response to include a considerable steady state error. This comes from the lack of the integral operator which reduces this unwanted behavior. The study in this paper aims to satisfy both stability and robustness of the plant with a FOPD controller by a new point of view. The contribution of this paper to the literature can be summarized as follows. The tuned controller is in FOPD type, which does not have the integral operator. This type of controller usually shows a response with steady state error. By the frequency frame tuning, the steady state error is relatively reduced. Also, by tuning gain and phase crossover frequencies while tuning the phase margin separately, the phase curve can be shaped without mathematical outgrowth. As the method is frequency domain based, frequency response formulas of a system with fractional order are comparatively complicated. Besides, calculating the derivative of the phase of the system at a certain frequency is a challenging issue. This paper uses a graphical approach to shape the phase of the system to reduce this complexity. A preliminary study on tuning FOPD controller for the FOPTD model with this new approach can be found in [40]. As the conclusion, frequency specifications of the system can easily be tuned via the frequency frame method and this provides a new point of view on robustness improvement issue without bringing new mathematical complexities. Also the method can successfully be used with different types of fractional order controllers.

The following sections of this paper are organized as follows. Section 2 represents FOPD controller, SOPTD plant and introduces the frequency frame. Third section gives the systematic design procedure of FOPD controller for SOPTD plant. Three illustrative examples are given in section 4 to clear the process. Section 5 has the conclusions.

2. SOPTD Plants, FOPD Controllers and the Frequency Frame

General representation of a SOPTD plant is given in the following form.

$$P(s) = \frac{K}{(T_1 s + 1)(T_2 s + 1)} e^{-Ls} \tag{1}$$

where, K is the gain, T_1 and T_2 are time constants, L is the time delay. The FOPD controller is defined in the following way.

$$C(s) = k_p + k_d s^{\mu}, \ \mu \in (0,2). \tag{2}$$

Thus, the system is,

$$G(s) = C(s)P(s). \tag{3}$$

The frequency frame is illustrated in Fig. 1. As known, the Bode plot consists of magnitude and phase curves. The frequency value when the magnitude curve cuts the $0dB$ line is called as the gain crossover frequency and shown as ω_{gc} in this paper. Similarly, the frequency value when the phase curve cuts the $-180°$ line is the phase crossover frequency and denoted as ω_{pc}. Difference of the magnitude plot with $0dB$ line at ω_{pc} is the gain margin (GM) and difference of the phase plot with $-180°$ line at ω_{gc} is the phase margin (PM). In this paper, controller parameters are tuned to ensure gain crossover frequency, phase crossover frequency and phase margin simultaneously. It has been widely studied in the last decades that these frequency properties have direct or indirect effects on the stability, performance and robustness of the controlled system. Tuning of phase and gain margins has been a challenging area of research and there can be found numerous studies in the literature. For example, independently selection of phase and gain margins based on the IMC structure is studied in [41]. Adjusting GM, PM, ω_{gc} and ω_{pc} based on frequency data is presented in [42].

In spite of the optimization techniques, the frequency frame method calculates the controller parameters analytically. One of the advantages of this is the reliability of the calculated parameters in that the parameters do not change in each computation. Comparing to optimization, analytical calculation of the parameters is relatively challenging. Optimization techniques are mostly heuristic and meta-heuristic approaches that approximate the result step by step. The frequency frame method finds the exact solution that provides the desired parameters.

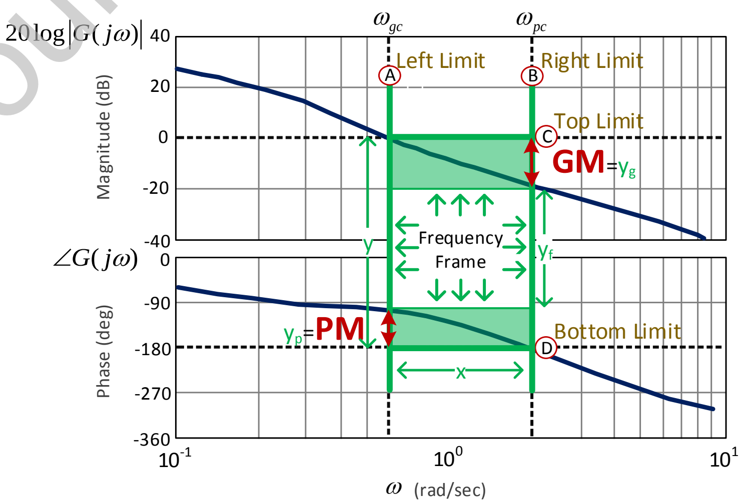


Fig. 1. The frequency frame.

As seen in Fig. 1, left limit of the frame is named as A and the right limit is named as B. Similarly, top and bottom limits of the frame are named as C and D respectively. Lengths of the upper and lower edges are determined as $x = \omega_{pc} - \omega_{gc}$. Lengths of the side edges are $y = y_p + y_f + y_g$ where, y_p is the phase margin and y_g is the gain margin. y_f is the remaining line between lower value of GM and upper value of PM. Purpose of the frequency frame is to shape the curves inside the frame by changing the lengths of related edges. This will provide us to enhance the margin of the system robustness. Tuning of phase and gain margins will be possible by straitening the edge y while fixing the limits A and B. Similarly, expanding the x edge while C and D limits are fixed, will give us the possibility to shape the magnitude and phase curves.

As referred in the first section, existing studies mostly consider only the gain crossover frequency to tune the controller. However, this method focuses on tuning the controller embracing both crossover frequencies. Therefore, the frequency frame method which includes both frequencies is developed. Next section presents the tuning procedure of FOPD controller for SOPTD plants.

3. FOPD Controller Design for SOPTD plants via the Frequency Frame

In this section, frequency properties mentioned in the previous sections are given. Then, the theorems to compute the controller are given. Following are the gain and phase specifications considered.

- Phase at ω_{gc},

$$\angle G(j\omega_{gc}) = PM - \pi \tag{7}$$

- Gain at ω_{gc},

$$\left|G(j\omega_{gc})\right| = 1 \tag{8}$$

- Phase at ω_{pc},

$$\angle G(j\omega_{pc})\Big| = -\pi \tag{9}$$

- Gain at ω_{pc},

$$\left|G(j\omega_{pc})\right| = 10^{GM/20} \tag{10}$$

Frequency frame is a frequency domain approach thus, frequency domain notations of both plant and the controller have to be found. Frequency domain response of the plant can be found by replacing s to $j\omega$ in Eq. (1) as,

$$\begin{aligned} P(j\omega) &= \frac{K}{\left(T_1(j\omega)+1\right)\left(T_2(j\omega)+1\right)} e^{-L(j\omega)} \\ &= \sqrt{\frac{K^2}{\left(1+T_1^2\omega^2\right)\left(1+T_2^2\omega^2\right)}} \angle -\arctan\left(\frac{K\left(T_1+T_2\right)\omega}{K-KT_1T_2\omega^2}\right) - L\omega \end{aligned} \tag{11}$$

Similarly, frequency response of the controller is,

$$C(j\omega) = k_p + k_d\left(j\omega\right)^{\mu} = k_p + k_d\omega^{\mu}\cos\left(\pi\mu/2\right) + jk_d\omega^{\mu}\sin\left(\pi\mu/2\right) \tag{12}$$

Thus, the system is,

$$G(j\omega) = C(j\omega)P(j\omega) \tag{13}$$

Then, following theorem holds on finding the FOPD parameters based on desired phase margin at the desired gain crossover frequency.

Theorem 1: Considering Eq. (7) and Eq. (8), desired PM at desired ω_{gc} can be provided by the following FOPD controller.

$$k_p = \pm\frac{\sqrt{1+T_1^2\omega_{gc}^2}\sqrt{1+T_2^2\omega_{gc}^2}}{K\sqrt{1+\tan\left(\varphi_1\right)^2}} \mp \frac{\sqrt{1+T_1^2\omega_{gc}^2}\sqrt{1+T_2^2\omega_{gc}^2}\cot\left(\pi\mu/2\right)\tan\left(\varphi_1\right)}{K\sqrt{1+\tan\left(\varphi_1\right)^2}} \tag{14}$$

and

$$k_d = \pm\frac{\omega_{gc}^{-\mu}\sqrt{1+T_1^2\omega_{gc}^2}\sqrt{1+T_2^2\omega_{gc}^2}\csc\left(\pi\mu/2\right)\tan\left(\varphi_1\right)}{K\sqrt{1+\tan\left(\varphi_1\right)^2}} \tag{15}$$

where,

$$\varphi_1 = PM - \pi + \arctan\left(\frac{K\left(T_1+T_2\right)\omega_{gc}}{K-KT_1T_2\omega_{gc}^2}\right) + L\omega_{gc} \tag{16}$$

Proof: Frequency domain notations of the plant and the controller was given in Eq. (11) and Eq. (12) respectively. Therefore, magnitude and phase of the plant and the controller are as follows.

$$\left|P(j\omega)\right| = \sqrt{\frac{K^2}{\left(1+T_1^2\omega^2\right)\left(1+T_2^2\omega^2\right)}} \tag{17}$$

$$\angle P(j\omega) = -\arctan\left(\frac{K\left(T_1+T_2\right)\omega}{K-KT_1T_2\omega^2}\right) - L\omega \tag{18}$$

$$\left|C(j\omega)\right| = \sqrt{\left(k_p + k_d\omega^{\mu}\cos\left(\pi\mu/2\right)\right)^2 + \left(k_d\omega^{\mu}\sin\left(\pi\mu/2\right)\right)^2} \tag{19}$$

$$\angle C(j\omega) = \arctan\left(\frac{k_d\omega^{\mu}\sin\left(\pi\mu/2\right)}{k_p + k_d\omega^{\mu}\cos\left(\pi\mu/2\right)}\right) \tag{20}$$

From above equations, magnitude and phase of the system are,

$$\left|G(j\omega)\right| = \left|C(j\omega)P(j\omega)\right| = \left|C(j\omega)\right|\left|P(j\omega)\right| \tag{21}$$

$$\angle G(j\omega) = \angle C(j\omega)P(j\omega) = \angle C(j\omega) + \angle P(j\omega) \tag{22}$$

Replacing ω with ω_{gc} in Eq. (21) with consideration of specification (ii) in Eq. (8) results as,

$$\begin{aligned}\left|G(j\omega_{gc})\right| &= \left|C(j\omega_{gc})\right|\left|P(j\omega_{gc})\right| = 1\\ &= \sqrt{\left(k_p + k_d\omega_{gc}^{\mu}\cos\left(\pi\mu/2\right)\right)^2 + \left(k_d\omega_{gc}^{\mu}\sin\left(\pi\mu/2\right)\right)^2} \times \sqrt{\frac{K^2}{\left(1+T_1^2\omega_{gc}^2\right)\left(1+T_2^2\omega_{gc}^2\right)}}\end{aligned} \tag{23}$$

Similarly, replacing ω with ω_{gc} in Eq. (22) with consideration of specification (i) in Eq. (7) results as,

$$\begin{aligned}\angle G(j\omega_{gc}) &= \angle C(j\omega_{gc}) + \angle P(j\omega_{gc}) = PM - \pi\\ &= \arctan\left(\frac{k_d\omega_{gc}^{\mu}\sin\left(\pi\mu/2\right)}{k_p + k_d\omega_{gc}^{\mu}\cos\left(\pi\mu/2\right)}\right) - \arctan\left(\frac{K\left(T_1+T_2\right)\omega_{gc}}{K-KT_1T_2\omega_{gc}^2}\right) - L\omega_{gc}\end{aligned} \tag{24}$$

According to the magnitude and phase in Eq. (23) and Eq. (24) k_p and k_d of the controller can be found as given in Theorem 1. □

Theorem 2: Considering Eq. (9) and Eq. (10), desired GM at desired ω_{pc} can be achieved with the following controller.

$$k_p = \pm \frac{10^{GM/20}\sqrt{1+T_1^{\,2}\omega_{pc}^2}\sqrt{1+T_2^{\,2}\omega_{pc}^2}}{K\sqrt{1+\tan(\varphi_2)^2}} \mp \frac{10^{GM/20}\sqrt{1+T_1^{\,2}\omega_{pc}^2}\sqrt{1+T_2^{\,2}\omega_{pc}^2}\cot(\pi\mu/2)\tan(\varphi_2)}{K\sqrt{1+\tan(\varphi_2)^2}} \tag{25}$$

and

$$k_d = \pm \frac{10^{GM/20}\omega_{pc}^{-\mu}\sqrt{1+T_1^{\,2}\omega_{pc}^2}\sqrt{1+T_2^{\,2}\omega_{pc}^2}\csc(\pi\mu/2)\tan(\varphi_2)}{K\sqrt{1+\tan(\varphi_2)^2}} \tag{26}$$

where,

$$\varphi_2 = -\pi + \arctan\left(\frac{K(T_1+T_2)\omega_{pc}}{K - KT_1T_2\omega_{pc}^2}\right) + L\omega_{pc} \tag{27}$$

Proof: Similar to the previous proof, replacing ω with ω_{pc} in Eq. (21) with consideration of specification (iv) in Eq. (10) results as,

$$\begin{aligned}\left|G(j\omega_{pc})\right| &= \left|C(j\omega_{pc})\right|\left|P(j\omega_{pc})\right| = 10^{GM/20} \\ &= \sqrt{\left(k_p + k_d\omega_{pc}^{\mu}\cos(\pi\mu/2)\right)^2 + \left(k_d\omega_{pc}^{\mu}\sin(\pi\mu/2)\right)^2} \times \sqrt{\frac{K^2}{\left(1+T_1^{\,2}\omega_{pc}^2\right)\left(1+T_2^{\,2}\omega_{pc}^2\right)}}\end{aligned} \tag{28}$$

Replacing ω with ω_{pc} in Eq. (22) with consideration of specification (iii) in Eq. (9) results as,

$$\begin{aligned}\angle G(j\omega_{pc}) &= \angle C(j\omega_{pc}) + \angle P(j\omega_{pc}) = -\pi \\ &= \arctan\left(\frac{k_d\omega_{pc}^{\mu}\sin(\pi\mu/2)}{k_p + k_d\omega_{pc}^{\mu}\cos(\pi\mu/2)}\right) - \arctan\left(\frac{K(T_1+T_2)\omega_{pc}}{K - KT_1T_2\omega_{pc}^2}\right) - L\omega_{pc}\end{aligned} \tag{29}$$

Then, common solution of Eq. (28) and Eq. (29) results with the equations given in Theorem 2. □

Consequently, parameters of two controllers are derived. Controller formulas in the theorems may seem relatively complicated. This comes from the frequency response computation of $(j\omega)^{\mu}$ and the time delay term in the plant. Here, μ can take an arbitrary real number. The controller gets a plain structure when the common fractional order μ is found. For this purpose, solutions of k_p in Eq. (14) and Eq. (25) as well as solutions of k_d in Eq. (15) and Eq. (26) have to be found in the range $\mu \in (0,2)$. Firstly, Eq. (14) and Eq. (25) have to be equalized and $10^{GM/20}$ w.r.t $\mu \in (0,2)$ have to be plotted. Similarly, equalization of Eq. (15) and Eq. (26) and plotting $10^{GM/20}$ will give the second plot. Intersection of the curves gives μ. Then, μ can be replaced in the related equations of k_p and k_d. There can be drawn two different plots due to the opposite signs of the equations as found previously. As the fractional order takes its values in the interval $\mu \in (0,2)$, the plot in the positive region is acceptable. This procedure is clearly explained on three examples in the next section.

4. Case Study

This section presents three examples to demonstrate the frequency frame. Stability analysis of the systems are realized by plotting the step responses for each controlled system.

Example 1: Consider the SOPTD plant provided from [43].

$$P_1(s) = \frac{0.3}{(2s+1)(s+1)} e^{-0.01s} \tag{30}$$

Desired gain and phase crossover frequencies for this example are $\omega_{gc} = 10 rad/s$ and $\omega_{pc} = 180 rad/s$. Suppose that $PM = 90^\circ$. By replacing the above variables in Eqs. (14) – (25) and Eqs. (15) – (26), k_p and k_d can be obtained w.r.t. μ. Then by plotting $10^{GM/20}$ in the range $\mu \in (0,2)$ we can obtain μ. Fig. 2 presents the related plot.

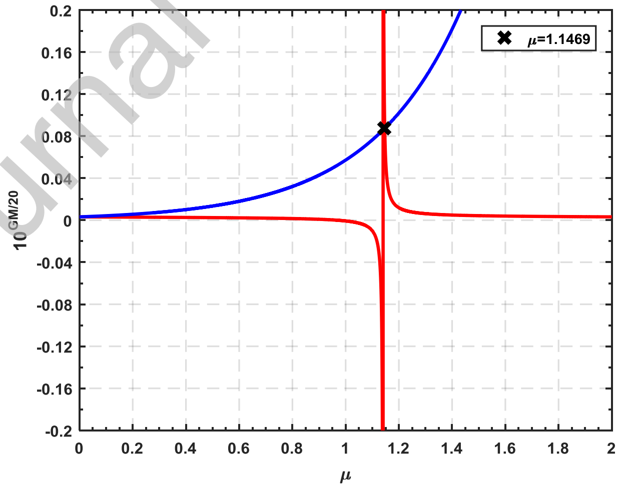


Fig. 2. Plot of $10^{GM/20}$ w.r.t $\mu \in (0,2)$.

Red curve is the plot of $10^{GM/20}$ obtained by equalizing Eq. (14) to Eq. (25) and blue curve is the plot of $10^{GM/20}$ obtained by equalizing Eq. (15) to Eq. (26) in the range of $\mu \in (0,2)$. Their intersection point shows the common fractional order $\mu = 1.1469$. Thus, following controller is obtained.

$$C_1(s) = 190.643 + 49.0765 s^{1.14686} \tag{31}$$

Bode plot of the system $G_1(s) = C_1(s)P_1(s)$ is shown in Fig. 3.

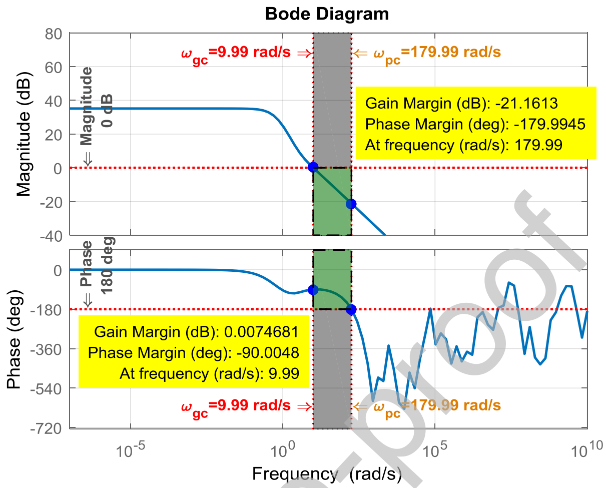


Fig. 3. Bode plot of the system $G_1(s) = C_1(s)P_1(s)$.

It can be seen in Fig. 3 that the desired crossover frequencies are satisfied. Fig. 4 illustrates the closed loop step response of the above system and also step responses of the system with $\pm 50\%$ iteration of k_p. We can conclude from the step response in Fig. 4 that the system is stable within obtained controller.

Systems could be under unexpected load disturbances. We can also test the proposed method on the system under disturbances as shown in Fig. 5. Fig. 6 presents the step response of the original system with $PM = 90^\circ$ and step response of the system with 10% load disturbance.

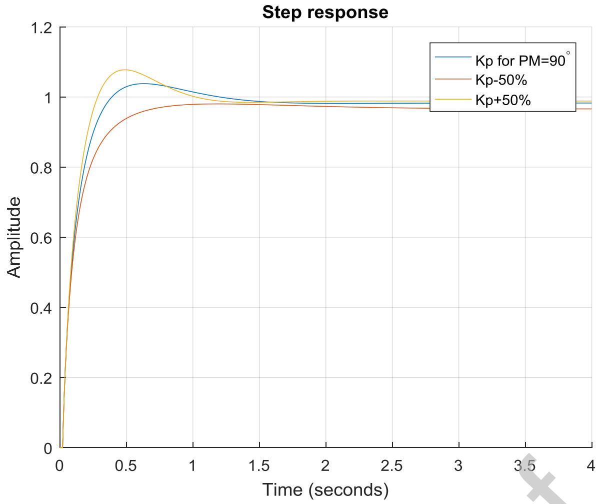


Fig. 4. Step responses of $G_1(s) = C_1(s)P_1(s)$ and $\pm 50\%$ variations of k_p.

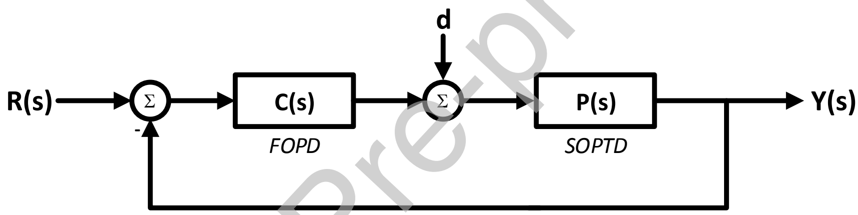


Fig. 5. SOPTD system under load disturbance

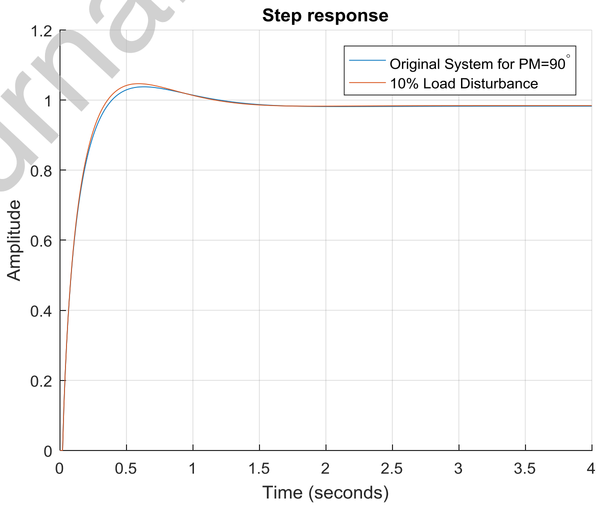


Fig. 6. Step response of the system under load disturbance of 10%.

According to Fig. 6, the system kept its stability against the load disturbance of 10%. We can compare the results obtained in Fig. 3 and Fig. 4 with existing studies. The study in [24] aims to tune FO[PD] controller for motion control system. The plant used in the paper is a FOPTD model and the aim is to tune the gain crossover frequency to be $\omega_{gc} = 1.2\, rad/\sec$ and the phase margin to be $PM = 60°$. When we check Fig. 3 in the paper, we can say that the given values are pretty approximated. In the cited paper, flatness of the curve is provided by equalizing the derivative of the phase to zero at ω_{gc}. Similarly, stability is shown by giving the step response of the system in Fig. 4. With the frequency frame method, not only the gain crossover frequency, both ω_{gc} and ω_{pc} specifications are achieved. In [24], robustness is checked by giving the step response of the system with $\pm 20\%$ variations of the time constant T in Fig. 7. With the help of the frequency frame method, the system remained stable against $\pm 50\%$ iterations of the controller gain k_p. Also it is shown that the system response almost remained the same under unexpected load disturbances. Let us consider the study in [20] which aims to tune FOPD controller for non-time delayed first order plant. Objective of the design is to satisfy desired gain crossover frequency and the phase margin. The interested specifications are set as $\omega_{gc} = 60\, rad/\sec$ and $PM = 70°$. Fig 2 in [20] shows that these specifications are successfully obtained. However, the phase crossover frequency and the gain margin properties are disregarded. Also, the robustness in Fig. 3 is satisfied by setting the phase derivative to zero at $\omega_{gc} = 60\, rad/\sec$. Thus, the frequency frame method brings a different point of view. The advantages of the frequency frame can also be investigated by comparing the results in [21-23].

Proposed equations can be tested with Bode plots of the system with varying PM values. The phase margin is varied in the range of $PM \in (10^o \to 90^o)$ with increment steps of 10^o. Then we obtained 9 different controllers for the SOPTD plant. Bode plots of these 9 systems are given in Fig. 7. Also step responses are given in Fig. 8.

Fig. 8 shows that all systems with changing PM values show stable behaviour. Also the systems are robust against gain variations of up to $\pm 50\%$. This proves the effect of the frequency frame. Now, another example of a SOPTD plant is considered.

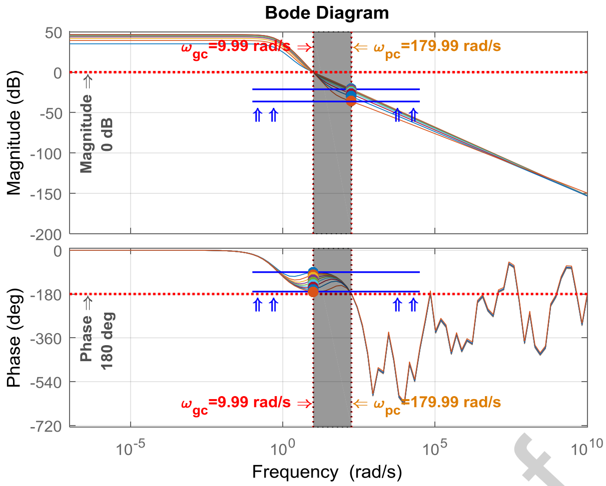


Fig. 7. Bode plots of the system with varying PM values

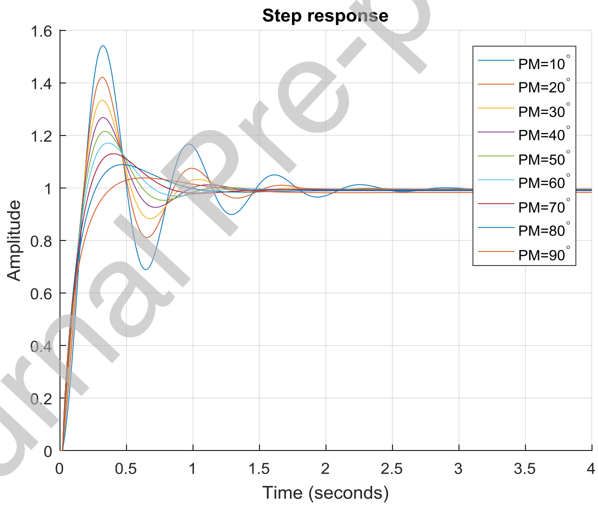


Fig. 8. Step responses of the systems with varying PM values.

Example 2: The following plant is provided from [44].

$$P_2(s) = \frac{100}{(s+1)(100s+1)}e^{-0.2s} \tag{32}$$

where, the crossover frequencies are desired to be $\omega_{gc} = 2.4 rad/s$ and $\omega_{pc} = 9.6 rad/s$. The phase margin for this system is considered to be in the range of $PM \in (10^o \to 90^o)$. Table 1 lists the controller parameters found for varying values of the PM with increment step of 10^o.

Table 1. k_p, k_d, μ and GM values found for the variations of PM.

PM	k_p	k_d	μ	GM
10°	7.41459	0.584873	1.45691	-17.5734
20°	7.42089	0.939277	1.37680	-14.1916
30°	7.15097	1.274590	1.32918	-12.0730
40°	6.63437	1.577470	1.29542	-10.6383
50°	5.89552	1.837250	1.26862	-9.66704
60°	4.96091	2.045370	1.24545	-9.05521
70°	3.86056	2.195290	1.22390	-8.75061
80°	2.62791	2.282650	1.20247	-8.73079
90°	1.29881	2.305470	1.17972	-8.99532

Considering the PM to be 30°, Bode plot of the plant in this case is given in Fig. 9. Similarly, step response of the system with $\pm 50\%$ iterations of the gain when $PM = 30°$ is illustrated in Fig. 10. Also step response of the system under 10% load disturbance is presented in Fig. 11. According to Fig. 10 and Fig. 11, stability of the system is successfully achieved. It is observed that the step response almost remained the same against the load disturbance. This proved the effectiveness of the frequency frame for SOPTD plants controlled with FOPD controllers.

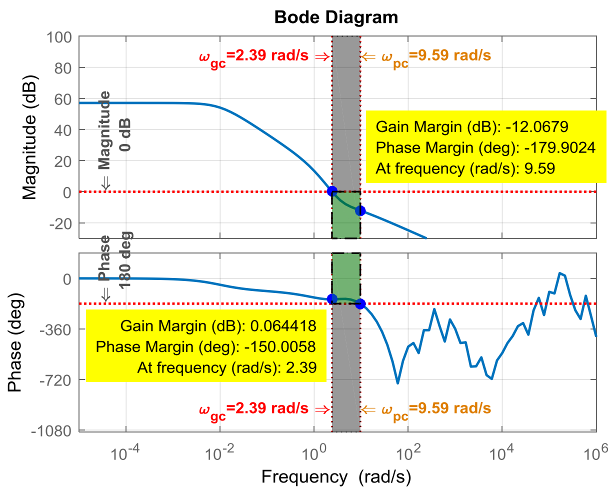


Fig. 9. Bode plot of the plant when $PM = 30°$

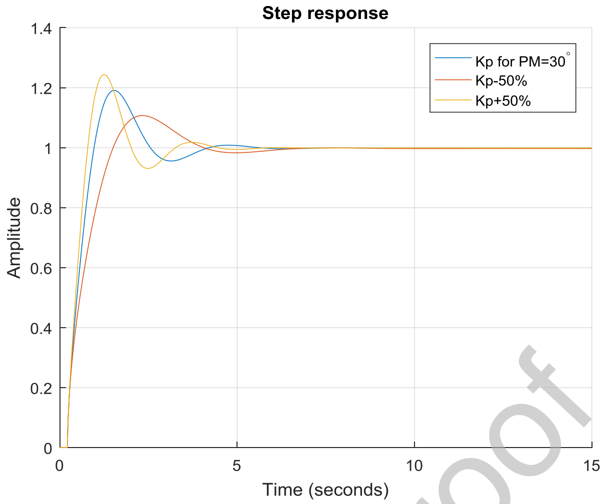


Fig. 10. Step response of the system with $\pm 50\%$ variations in the gain.

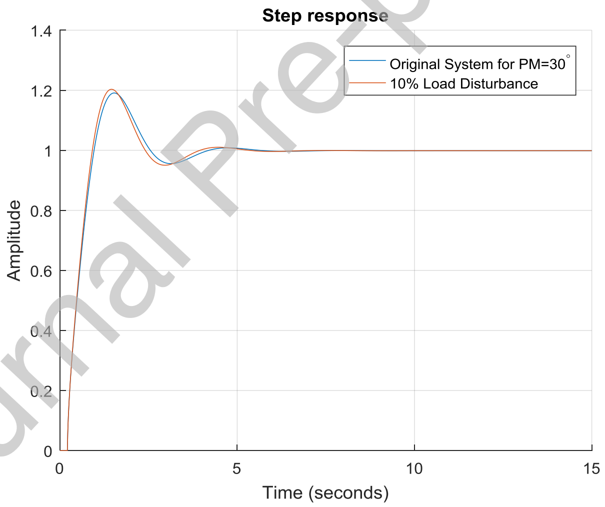


Fig. 11. Step response of the system under 10% load disturbance.

Let us study on an SOPTD plant with high steady state error.

Example 3: The following plant is provided from [45].

$$P_3(s) = \frac{0.5}{(s+1)(0.5s+1)} e^{-1.64s} \tag{33}$$

We consider the gain and phase margins to be $\omega_{gc} = 0.3 rad/s$ and $\omega_{pc} = 0.9 rad/s$ and the phase margin to be $PM = 30°$. Thus, following controller is obtained.

$$C_3(s) = 4.703110 + 39.1087 s^{1.72020} \tag{34}$$

Bode plot of the system controlled with the controller above is given in Fig. 12.

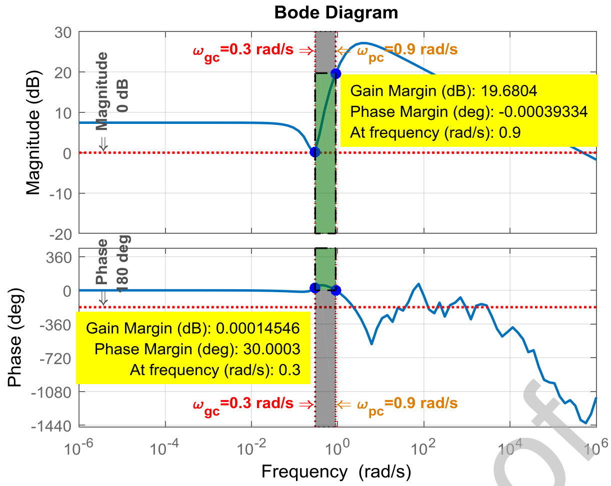


Fig. 12. Bode plot of the system $C_3(s)P_3(s)$.

Desired ω_{gc} , ω_{pc} and PM are successfully satisfied. Also, step response of the system is presented in Fig. 13. Fig. 13 shows that the system response is stable but it has a steady state error as we expect. We can form the plant to include an integrator as $P_3(s)/s$ and compute the related controller equations for the new plant. Then we obtain the step response given in Fig. 14. It can be seen that the steady state error is considerably reduced. Consequently, the controller obtained with the frequency frame method are proved with different examples.

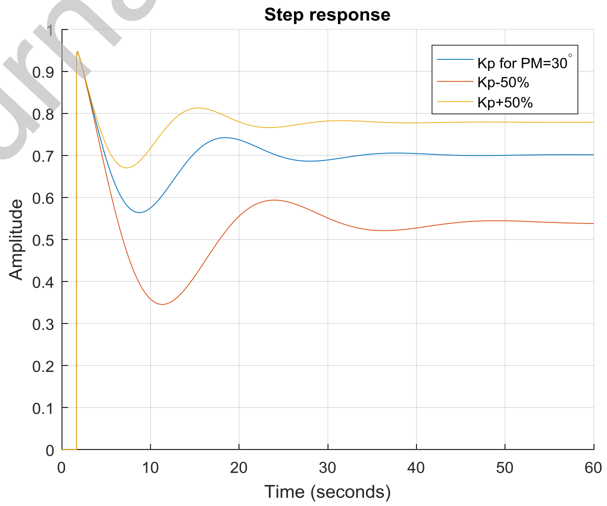


Fig. 13. Step response of the system $C_3(s)P_3(s)$.

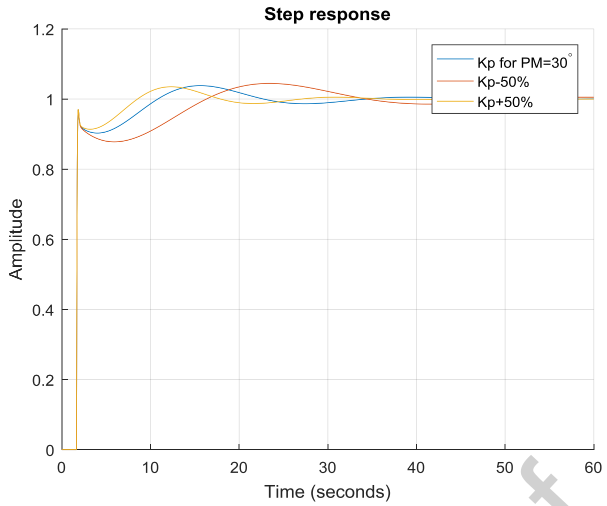


Fig. 14. Step response of the system including an integrator.

5. Conclusion

This paper performs the frequency frame approach for performance and robustness of SOPTD plants by tuning FOPD controllers. It is observed that the controller obtained with the method provided improved stability and robustness for such plants. Varied from existing studies, the frequency frame approach considers both gain and phase crossover frequencies when computing the controller parameters. With the help of the method, the system remained stable against $\pm 50\%$ iterations of the controller gain k_p. Also it is shown that the system response almost remained the same under unexpected load disturbances.Thus, desired results are satisfactorily obtained and mathematical complexity is considerably reduced. The method is verified with three examples.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] K.B. Oldham, J. Spanier, Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order, first ed., Academic Press, New York–London, 1974.
[2] R.E. Gutiérrez, J.M. Rosário, J.A.T. Machado, Fractional order calculus: basic concepts and engineering applications, Math. Probl. in Eng. (2010) 1-19. https://doi.org/10.1155/2010/375858.
[3] L. Debnath, A brief historical introduction to fractional calculus, Int. J. of Math. Educ. in Sci. and Technol. 35 (4) (2004) 487–501. https://doi.org/10.1080/00207390410001686571.

[4] I. Petras, Stability of fractional-order systems with rational orders: a survey, Fract. Calc. and Appl. Anal. 12 (3) (2009) 269-298.

[5] F.J.V. Parada, J.A.O. Tapia, J.A. Ramirez, Effective medium equations for fractional Fick's law in porous media, Physica A 373 (2007) 339–353. https://doi.org/10.1016/j.physa.2006.06.007.

[6] P. Arena, R. Caponetto, L. Fortuna, D. Porto, Nonlinear non-integer order circuits and systems – an introduction, World Scientific, Singapore, 2000.

[7] B.T. Krishna, K.V.V.S. Reddy, Active and passive realization of fractance device of order ½, Act. and Passive Electron. Compon. (2008) 1-5. https://doi.org/10.1155/2008/369421.

[8] B.M. Vinagre, Y.Q. Chen, I. Petras, Two direct Tustin discretization methods for fractional-order differentiator / integrator, J. of the Franklin Inst. 340 (5) (2003) 349–362. https://doi.org/10.1016/j.jfranklin.2003.08.001.

[9] M.F. Silva, J.A.T. Machado, A.M. Lopes, Fractional order control of a hexapod robot, Nonlinear Dyn. 38 (1-4) (2004) 417–33. https://doi.org/10.1007/s11071-004-3770-8.

[10] M.F.M. Lima, J.A.T. Machado, M. Crisóstomo, Experimental signal analysis of robot impacts in a fractional calculus perspective, J. of Adv. Comp. Intell. and Intell. Inf. 11 (9) (2007) 1079-1085.

[11] H. Bao, J.H. Park, J. Cao, Non-fragile state estimation for fractional-order delayed memristive BAM neural networks, Neural Netw. 119 (2019) 190–199. https://doi.org/10.1016/j.neunet.2019.08.003.

[12] H. Bao, J. Cao, J. Kurths, A. Alsaedi, B. Ahmad, H∞ state estimation of stochastic memristor-based neural networks with time-varying delays, Neural Netw. 99 (2018) 79–91. https://doi.org/10.1016/j.neunet.2017.12.014.

[13] H. Bao, J. Cao, J. Kurths, State estimation of fractional-order delayed memristive neural networks, Nonlinear Dyn. 94 (2018) 1215–1225. https://doi.org/10.1007/s11071-018-4419-3.

[14] B. Senol, A. Ates, B.B. Alagoz, C. Yeroglu, A numerical investigation for robust stability of fractional-order uncertain systems, ISA Trans. 53 (2) (2014) 189-198. https://doi.org/10.1016/j.isatra.2013.09.004.

[15] Z. Liao, C. Peng, W. Li, Y. Wang, Robust stability analysis for a class of fractional order systems with uncertain parameters, J. of the Franklin Inst. 348 (6) (2011) 1101-1113. https://doi.org/10.1016/j.jfranklin.2011.04.012.

[16] R. Matušů, B. Şenol, L. Pekař, Robust Stability of Fractional-Order Linear Time-Invariant Systems: Parametric versus Unstructured Uncertainty Models, Complexity (2018) 1-12. https://doi.org/10.1155/2018/8073481.

[17] A. Ates, C. Yeroglu, B.B. Alagoz, B. Senol, Tuning of fractional order PID with master-slave stochastic multi-parameter divergence optimization method, Int. Conf. on Fract. Diff. and Its Appl., Catania, Italy, 2014. https://doi.org/10.1109/ICFDA.2014.6967388.

[18] I. Podlubny, Fractional-order systems and PIλDμ-controllers, IEEE Trans. on Autom. Cont. 44 (1) (1999) 208–214. https://doi.org/10.1109/9.739144.

[19] S. Tufenkci, B. Senol, B.B. Alagoz, Stabilization of Fractional Order PID Controllers for Time-Delay Fractional Order Plants by Using Genetic Algorithm, 2018 Int. Conf. on Artif. Intell. and Data Proces., Malatya, Turkey, 2018.

[20] H. Li, Y.Q. Chen, A fractional order proportional and derivative (FOPD) controller tuning algorithm, Chinese Cont. and Decis. Conf., Yantai, China, 2008.

[21] H. Li, Y. Luo, Y.Q. Chen, A Fractional Order Proportional and Derivative (FOPD) Motion Controller: Tuning Rule and Experiments, IEEE Trans. on Cont. Syst. Technol. 18 (2) (2010) 516-520. https://doi.org/10.1109/TCST.2009.2019120.

[22] M. Zhang, X. Lin, W. Yin, An improved tuning method of fractional order proportional differentiation (FOPD) controller for the path tracking control of tractors, Biosyst. Eng. 116 (4) (2013) 478-486. https://doi.org/10.1016/j.biosystemseng.2013.10.001.

[23] L. Liu, S. Zhang, D. Xue, Y.Q. Chen, General robustness analysis and robust fractional-order PD controller design for fractional-order plants, IET Cont. Theory & Appl. 12 (12) (2018) 1730-1736. https://doi.org/10.1049/iet-cta.2017.1145.

[24] Y. Jin, Y.Q. Chen, D. Xue, Time-constant robust analysis of a fractional order [proportional derivative] controller, IET Cont. Theory & Appl. 5 (1) (2011) 164-172. https://doi.org/10.1049/iet-cta.2009.0543.

[25] C. Wang, L. Wang, M. Li, Application research for hydraulic servo system based on fractional order proportional derivative and [proportional derivative] controllers, Proc. of the 33rd Chinese Cont. Conf., Nanjing, China, 2014.

[26] J. Yang, L. Dong, X. Liao, Fractional order PD controller based on ADRC algorithm for DC motor, IEEE Conf. and Expo Transp. Electr. Asia-Pacific., Beijing, China, 2014.

[27] V. Badri, M.S. Tavazoei, Achievable Performance Region for a Fractional-Order Proportional and Derivative Motion Controller, IEEE Trans. on Ind. Electron. 62 (11) (2015) 7171-7180. https://doi.org/10.1109/TIE.2015.2448691.

[28] S. Padhee, Controller Design for Temperature Control of Heat Exchanger System: Simulation Studies, WSEAS Trans. on Syst. and Cont. 9 (2014) 485-491.

[29] S. Santosh, M. Chidambaram, Tuning of Proportional Integral Derivative Controllers for Critically Damped Second-Order Plus Time Delay Systems, Indian Chem. Eng. 57 (1) (2015) 32-51. https://doi.org/10.1080/00194506.2014.975760.

[30] Bajarangbali, S. Majhi, Modeling of stable and unstable second order systems with time delay, Annu. IEEE India Conf., Mumbai, India, 2013.

[31] M. Irshad, A. Ali, A review on PID tuning rules for SOPTD inverse response processes, Int. Conf. on Intell. Comp., Instrum. and Cont. Technol., Kannur, India, 2017.

[32] R.P. Wang, Y.G. Pi, Fractional order proportional and derivative controller design for second-order systems with pure time-delay, Int. Conf. on Mechatron. Sci., Electr. Eng. and Comp., Jilin, China, 2011.

[33] C. Wang, Y. Luo, Y.Q. Chen, Fractional order proportional integral (FOPI) and [proportional integral] (FO[PI]) controller designs for first order plus time delay (FOPTD) systems, Chinese Cont. and Decis. Conf., Guilin, China, 2009.

[34] Y. Luo, Y.Q. Chen, Fractional-order [proportional derivative] controller for robust motion control: Tuning procedure and validatio, American Cont. Conf., St. Louis, USA, 2009.

[35] C. Wang, Y. Jin, Y.Q. Chen, Auto-tuning of FOPI and FO[PI] controllers with iso-damping property, Conf. on Decis. and Cont., Shanghai, China, 2009.

[36] C.A. Monje, B.M. Vinagre, Y.Q. Chen, V. Feliu, P. Lanusse, J. Sabatier, Proposals for fractional PID tuning, IFAC Symp. on Fract. Diff. and its Appl., Bordeaux, France, 2004.

[37] Y.Q. Chen, K.L. Moore, B.M. Vinagre, I. Podlubny, Robust PID controller autotuning with a phase shaper, IFAC Workshop on Fract. Diff. and its Appl., Bordeaux, France, 2004.

[38] S. Saha, S. Das, R. Ghosh, B. Goswami, R. Balasubramanian, A.K. Chandra, S. Das, A. Gupta, Fractional order phase shaper design with Bode's integral for iso-damped control system, ISA Trans. 49 (2) (2010) 196-206. https://doi.org/10.1016/j.isatra.2009.12.001.

[39] B. Şenol, U. Demiroğlu, Frequency frame approach on loop shaping of first order plus time delay systems using fractional order PI controller, ISA Trans. 86 (2019) 192-200. https://doi.org/10.1016/j.isatra.2018.10.021.

[40] B. Şenol, U. Demiroğlu, Fractional Order Proportional Derivative Control for First Order Plus Time Delay Plants: Achieving Phase and Gain Specifications Simultaneously, Trans. of the Inst. of Meas. and Cont. 41 (5) (2019) 4358-4369. https://doi.org/10.1177/0142331219857397.

[41] P.P. Arya, S. Chakrabarty, A Modified IMC Structure to Independently Select Phase Margin and Gain Cross-over Frequency Criteria, IFAC-PapersOnLine 50 (1) (2018) 267-272. https://doi.org/10.1016/j.ifacol.2018.05.066

[42] N. Sayyaf, M.S. Tavazoei, Desirably Adjusting Gain Margin, Phase Margin, and Corresponding Crossover Frequencies Based on Frequency Data, IEEE Trans. on Ind. Inf. 13 (5) (2017) 2311-2321. https://doi.org/10.1109/TII.2017.2681842.

[43] C. Rajapandiyan, M. Chidambaram, Closed-Loop Identification of Second-Order Plus Time Delay (SOPTD) Model of Multivariable Systems by Optimization Method, Ind. & Eng. Chem. Res. 51 (28) (2012) 9620-9633. https://doi.org/10.1021/ie203003p.

[44] Z. Zhao, Z. Liu, J.J. Zhang, IMC-PID tuning method based on sensitivity specification for process with time-delay, J. of Cent. South Univ. Technol. 18 (4) (2011) 1153-1160. https://doi.org/10.1007/s11771-011-0817-0.

[45] S. Srivastava, V.S. Pandit, A PI/PID controller for time delay systems with desired closed loop time response and guaranteed gain and phase margins, J. of Process Cont. 37 (2016) 70-77. https://doi.org/10.1016/j.jprocont.2015.11.001.