Marketing effort within the newsvendor problem framework: A systematic review and extensions of demand-effort and cost-effort formulations

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Marketing effort within the newsvendor problem framework: A systematic review and extensions of demand-effort and cost-effort formulations

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Abstract

This study deals with a generalization of the newsvendor problem with marketing efforts. Combining inventory and marketing decisions is currently a topic that has been widely studied. Various formulations of marketing efforts and their costs have been applied in the literature. Therefore, a systematic review was performed to identify existing formulations, especially those that deal with mathematical modeling and optimization. The findings observed on a wide set of marketing effort formulations are summarized, generalized, and applied in the newsvendor problem framework. It was found that the optimal marketing effort decision does not depend on the uncertainty involved in the model for the additive demand case under commonly used assumptions. Optimal marketing is equal to its deterministic equivalent, contrary to the multiplicative form, where the decision directly depends on uncertainty. Formulations of the demand-effort response function and the cost of marketing effort are generalized (to concave and convex functions, respectively) and extended to an S-shaped demand-effort response function. Assumptions and theorems that guarantee the uniqueness of optimal marketing efforts are established. Finally, the effects of price and cost parameter changes on optimal marketing effort decisions were analyzed.

Keywords: marketing effort, newsvendor problem, systematic review, inventory optimization, additive demand, multiplicative demand

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1. Introduction

The coordination between marketing and inventory decisions has been widely studied in recent decades. Several producers have used innovative marketing strategies to effectively control their inventories [1]. In particular, the rapid development of e-commerce in the retail industry has made marketing efforts a crucial aspect of various inventory optimization problems [2]. However, it has become increasingly difficult to assess the effectiveness of investments across marketing channels [3].

Mathematical optimization and modeling are widely used techniques for optimizing market- and demand-related problems in various application areas. An essential issue in marketing is to understand the impact of various marketing efforts on sales and revenue. In addition to many typical inventories and revenue-related optimization problems, the initiatives taken in relation to the following industries and applications can be cited: market dynamics and reverse logistics for sustainability in the pharmaceutical industry [4], conceptual circular economy [5], and conceptual waste management planning [6]. Increasing attention has led to a broad set of mathematical formulations for marketing efforts within inventory-optimization models.

Demand uncertainty is inherent in almost all practical business environments and has been studied extensively in inventory management literature [7]. The widespread use of various marketing strategies and their formulations makes the environment even more complex. Various advertising response functions were reviewed in [8, 1]. However, modern approaches for coordinating inventory and marketing strategies handle the marketing in a broader sense: appropriate marketing activities span, e.g., providing shelf spaces, promotional displays, advertising, after-sales service support, and other demand-enhancing activities [9]. Generally, studies dealing with simple advertising decisions define advertising costs as a simple linear function. However, the cost of a more general marketing effort is described by various (linear or non-linear) functions. An example can be provided by [7], where the consideration of a linear demand-effort response function and convex cost of marketing effort concludes that a product with a lower marketing cost function always benefits more from improved supply reliability than a product with a higher marketing cost function. Therefore, articles that provide further insights into the coordination of marketing decisions in uncertain (stochastic) models are of great value.

This paper aims to summarize existing formulations, generalize them, and extend them. The systematic review approach was used to collect existing articles dealing with optimizing marketing efforts and inventory decisions, especially of the newsvendor-like simple framework. Based on this, a review of the formulations of the demand-effort response function as a function of marketing effort, and the related cost of the marketing effort is provided. In the last decade, researchers have focused more on S-shaped demand functions, which are believed to be broadly applicable in the industry [8]. Therefore, this study extends the existing results on the

S-shaped demand-effort response function, which presents a more practical market behavior dependency (see, e.g., [1]), combined with the non-linear marketing effort function.

The remainder of this paper is organized as follows. In Section 2, a systematic review of relevant studies involving mathematical models that combine inventory and marketing effort decisions is presented. Then, the findings from Section 2 are applied and generalized to the newsvendor problem with marketing efforts in Section 3. Finally, the conclusions of this study are summarized in Section 4.

2. Systematic review

To provide a systematic review of marketing effort formulations, the following research questions were defined:

1. What are the commonly used formulations of marketing efforts?
2. Can the reviewed formulations and observations be generalized and unified?

2.1. Methodology and search strategy

To minimize the threat of missing relevant papers, the Scopus database was searched using the following criteria (aimed at article title, abstract, and keywords):

- (marketing `AND` effort)

  `AND`

- [newsvendor`OR` newsboy `OR` (stochastic `AND` single `AND` period)].

The search led to 18 published articles (only papers written in English were included) that were reviewed. Their assessments are summarized in Table 1. During the review, more specific (inclusion) criteria were applied. The final phase includes the following steps.

- combine inventory and marketing effort decisions in single modeling and optimization (newsvendor-like) framework;
- include a mathematical description (either a mathematical expression or at least general assumptions) of the marketing effort;
- the marketing effort cost is not defined in a linear form (i.e., not simple advertising, see, e.g., [1]).

Table 1: Systematic review: Assessment of the found references (Scopus database, February 4, 2022)

Year	Article	Newsvendor-like model	Marketing effort			Additional decisions
			Involved	Math. form	Exerted by	
1996	[10]	×	×	×	–	future promotions
2008	[8]	✓	✓	✓	retailer	market selection
2009	[11]	✓	✓	✓	retailer	supplier's output
2010	[12]	✓	✓	×	retailer	forecasting
2011	[13]	✓	✓	×	retailer	pricing
2013	[14]	✓	✓	×	salesforce	salesforce compensation
2014	[15]	✓	✓	✓	retailer & manufacturer	pricing
2014	[16]	×	✓	×	retailer	–
2015	[9]	✓	✓	✓	retailer	pricing
2015	[17]	✓	✓	✓	retailer	–
2015	[18]	✓	✓	✓	retailer & manufacturer	pricing, channel choice
2015	[19]	✓	✓	✓	retailer	rebate decision
2016	[20]	✓	✓	✓	retailer	–
2017	[21]	✓	✓	×	retailer & manufacturer	–
2018	[2]	✓	✓	✓	retailer	price discount
2018	[22]	✓	✓	✓	retailer	pricing
2020	[23]	✓	✓	✓	retailer	pricing (supplier)
2021	[24]	✓	✓	✓	retailer	pricing

2.2. Study selection

The full search and review process for the articles is summarized in Figure 1. The screening phase led to the exclusion of six papers for the reasons marked in Table 1 (with a cross ×). During the review, 12 relevant papers were found (by searching through, for example, the cited literature, citations of the papers, or other research of the authors of the articles) and added. In total, 20 identified papers were found to be relevant.

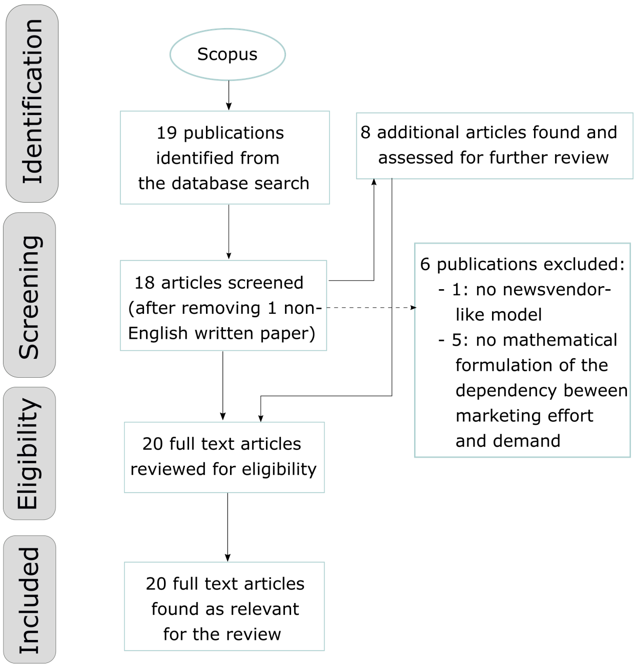

Figure 1: PRISMA flowchart [25] of references collection methodology

2.3. Findings of the systematic review

2.3.1. Review and historical remarks: The newsvendor problem with marketing effort

Optimization problems involving joint marketing (or advertising) and inventory decisions have been broadly investigated in recent decades. Advertising presents a term that is commonly used for a linear case of marketing effort cost; however, this is not the subject of this study. In addition, the first research papers dealing with the newsvendor model with joint marketing effort and quantity decisions attempted to provide general marketing effort formulations, followed by simple linear marketing effort cost formulations [26, 8]. Specifically, [26] defined a newsvendor-like model with diminishing returns on effort. [8] formulated the newsvendor problem as a non-linear integer optimization problem integrating market selection, marketing effort, and procurement decisions. Further, [27] and [28] defined the marketing effort cost as convex and quadratic, respectively, with linear demand-effort dependency. In the next decade, various problem formulations with various decisions, linear or non-linear demand effort, and marketing effort cost formulations were investigated; these are reviewed in Table 2,and other mathematical properties are summarized in Section 2.3.2.

Table 2: A summary of problem formulations: involved decisions and main properties

Year	Article	Decisions				Additional description
		Inventory	Marketing effort	Pricing	Additional	
1987	[26]	✓	✓			
2004	[27]	✓	✓		contracts	dual channel
2006	[28]	✓	✓		contracts	dual channel (sale timing)
2008	[8]	✓	✓			market selection (non-linear integer problem)
2009	[29]	✓	✓	✓	contracts	dual channel
2009	[11]	✓	✓		contracts	dual channel (drop-shipping)
2010	[7]	✓	✓			effects of supply reliability
2011	[30]	✓	✓	✓		selling cost proportional to the quantity
2013	[31]	✓	✓		contracts	return policy and discounts on whole sale price
2014	[15]	✓	✓	✓		single channel, dual channel
2015	[9]	✓	✓	✓		risk-aversion
2015	[17]	✓	✓			risk-aversion
2015	[18]	✓	✓	✓	contracts	dual channel
2015	[19]	✓	✓	✓	contracts	dual channel, quantity discount
2016	[20]	✓	✓			multi period, multi product
2017	[32]	✓	✓	✓	collection rate	pricing: retail price and whole sale price
2018	[2]	✓	✓	✓		pricing: price discount, advance selling
2018	[22]	✓	✓	✓		multi period
2020	[23]	✓	✓		contracts	dual channel
2021	[24]	✓	✓	✓		risk-aversion

2.3.2. Summary of mathematical formulations

The search strategy described above involves searching within the literature on the newsvendor (alternatively newsboy) problem or the so-called (stochastic) single-period problem. This type of problem(s) often serves as an illustrative decision-making framework in the literature dealing with inventory optimization, marketing and pricing optimization, or risk analysis. Herein, newsvendor-like models serve to investigate research and modeling ideas on marketing effort decision-making, which is often made under some degree of uncertainty.

The newsvendor-like models typically involve random elements that are related to the demand function. Among the 20 relevant studies, 19 involved a model with a random factor influencing demand. The random factor is typically captured via the additive or multiplicative demand form (five papers define the demand only in the additive form, 10 in the multiplicative form, and 3 in both forms). Although the multiplicative demand form is more frequent than the additive form (13 vs. 8), both types of random demand forms are commonly used, even though the relevance of the additive demand form is sometimes questioned (see, e.g., [1] for its discussion in the case of the newsvendor problem with advertising). For the additive demand form, three cases are considered: linear (5x), linear fractional, and concave (both 1x). Six cases are considered for the multiplicative form: linear and concave (both 4x), non-positive elasticity, power, linear fractional and diminishing returns on effort (all 1x). The marketing effort cost is defined in four various forms: quadratic (9x), convex (6x), linear (2x, which, on the other hand, can correspond to the classical advertising formulation), and power (1x).

The 20 papers found as relevant studies are reviewed in Table 3. For a more detailed summary of mathematical formulations and examples, see Table A.1 in Appendix A.

Table 3: A summary of selected publications based on random demand, random demand form, and mathematical form of the demand-effort, and marketing cost functions

Article	Random	Random demand form		Basic assumptions	
	demand	Additive	Multiplicative	Demand-effort function	Marketing effort cost
[26]	✓		✓	diminishing returns on effort	increasing
[27]	✓		✓	linear	convex
[28]	✓	✓		linear	quadratic
[8]	✓			concave	mean and variance (linear)
[29]	✓	✓		linear	convex
[11]	✓	✓		concave	convex
[7]	✓		✓	linear	convex
[30]	✓		✓	non-positive elasticity	convex
[31]	✓	✓	✓	linear fractional	not specified
[15]	✓		✓	power	convex
[9]	✓		✓	concave	convex
[17]	✓		✓	concave	convex
[18]	✓		✓	linear	convex
[19]	✓		✓	linear	convex
[20]	✓	✓		linear	not specified
[32]				linear	not specified
[2]	✓	✓		linear	not specified
[22]	✓	✓	✓	A: linear, M: concave	convex
[23]	✓		✓	concave	convex
[24]	✓	✓	✓	increasing	convex
# 20 articles	19	8	13		

Both types of random demand forms are used in the articles; thus, both are further considered and analyzed. Under certain additional assumptions concerning the demand-effort response function and marketing effort cost considered in some articles, all the considered formulations can be unified, as the demand-effort response function is increasing and concave in the marketing effort, and the marketing effort cost is increasing, continuously differentiable, and convex in the marketing effort.

3. Newsvendor problem with marketing effort

The newsvendor problem with the marketing effort considered herein can be described as follows. The newsvendor exerts an effort e (e.g., hours, days, and employees [8]) to increase the demand for the product with the total marketing effort cost $C(e)$. Simultaneously, he buys and stocks x units of product at a unit cost of c. Demand, denoted by $D(e, \xi)$, depends on marketing effort and is influenced by the random factorξ. If demand D is greater than the order quantity x, all stocked units are sold for revenue px, where p is the unit price, $p > c$. In this case, a loss given by a unit shortage penalty cost s is considered for all shortages $D - x$. Otherwise, if D is less than or equal to x, the revenue is only pD, and the leftovers, $x - D$, are salvaged through a unit salvage value v, $v < c$. The objective function, denoted by $\pi(e, x, \xi)$, is defined as

$$\pi(e,x,\xi) = \begin{cases} px - cx - s[D(e,\xi) - x] - C(e), & \text{for } x < D(e,\xi), \\ pD(e,\xi) - cx + v[x - D(e,\xi)] - C(e), & \text{for } x \geq D(e,\xi). \end{cases} \quad (1)$$

Let demand function $D(e,\xi)$ be given by the following formula [1]:

$$D(e,\xi) \equiv D(e,\xi_m,\xi_a) = d(e)\xi_m + \xi_a, \quad (2)$$

where ξ is a vector $\xi = (\xi_a, \xi_m)$, ξ_a and ξ_m are the so-called additive and multiplicative random variables, respectively, and the demand-effort response function $d(e)$ captures the deterministic dependency between the demand and marketing effort. Let $F(\cdot)$be the cumulative distribution function (cdf) of a random variable. We assume that the higher the marketing effort, the higher the demand. The demand is given by the function $d(e)$, which is supposed to be non-negative, continuous, increasing, and twice differentiable [9, 29]. Similarly, the function $C(e)$ is supposed to be convex, increasing, and continuously differentiable in e (see Table A.1 and section 2.3.2).

Furthermore, two forms of the demand function are considered: additive and multiplicative. To define the multiplicative demand case, let the additive random variable satisfy $\xi_a = 0$, and the expectation of the multiplicative random variable be $E[\xi_m] = 1$. In the additive demand case, we let $\xi_m = 1$ and $E[\xi_a] = 0$. Then, for both cases, the expectation of D is specified as $E[D(e,\xi_a,\xi_m)] = d(e)$.

3.1. Additive demand model

Let the demand function (2) be defined in an additive form, that is, $D_A(e,\xi_a) = d(e) + \xi_a$. Considering model (1), the additive demand form, and defining the stocking factor $z \in \mathbb{R}$ as $z = x - d(e)$ (if $x = 0$ then $e = 0$ but $d(0) > 0$), the expected profit can be rewritten as

$$\Pi(e,z) = \Psi(e) - l(z), \quad (3)$$

where $\Psi(e) = (p-c)d(e) - C(e)$ is the riskless profit, which would occur in the absence of uncertainty, and the so-called expected loss is $l(z) = (c-v)\Lambda(z) + (p+s-c)\Theta(z)$, where $\Lambda(z)$ denotes expected leftovers and $\Theta(z)$ expected shortages (see [1]).

From expression (3), we can see that decisions on e and z are made independently, unlike in the multiplicative model. Therefore, for the additive demand model, the optimal marketing effort e^* is always equal to the optimal riskless marketing effort e^*_Ψ and is given by the condition

$$(p-c)\frac{\mathrm{d}d(e^*)}{\mathrm{d}e} - \frac{\mathrm{d}C(e^*)}{\mathrm{d}e} = 0, \quad (4)$$

whereas the optimal stocking quantity z^* corresponds to the solution of the well-known classical newsvendor problem

$$z^* = F^{-1}\left(\frac{p+s-c}{p+s-v}\right) \quad (5)$$

under the assumption that F is invertible. The definition of the stocking factor then provides the optimal ordering amount as $x^* = z^* + d(e^*)$.

Example 1 (Optimal marketing effort for the linear demand case). Let the function $d(e)$ be linear, $d(e) = a + be$, and let marketing effort cost be quadratic, $C(e) = \frac{\mu e^2}{2}$, where $a, b, \mu > 0$. Then, substituting $d(e)$ and $C(e)$into condition (4), we obtain the optimal marketing effort, $e^* = \frac{(p-c)b}{\mu}$.

Example 2 (Optimal marketing effort for the concave demand case). Let the function $d(e)$ be given as $d(e) = ae^b$, and let marketing effort cost be quadratic, $C(e) = \frac{\mu e^2}{2}$, where $a, \mu > 0$ and $0 < b < 1$. Then, substituting $d(e)$ and $C(e)$into condition (4), we obtain the optimal marketing effort, $e^* = \left\{\frac{[p-c]ab}{\mu}\right\}^{\frac{1}{2-b}}$.

Theorem 1. *Let functions $d(e)$ and $C(e)$ satisfy additive model assumptions, $p - c > 0$, the demand-effort response function $d(e)$ be increasing and concave, and the cost of marketing effort $C(e)$ be increasing and convex. Then, the expected profit $\Pi(e,z^*)$ is concave in e, the globally optimal marketing effort e^* is unique, and if positive, it is given by the solution of (4).*

Proof. The concaveness of $\Pi(e,z^*)$ directly follows from the fact that $-C(e)$ is concave whenever $C(e)$ is convex, and concavity is an additive property. The uniqueness and determination of the globally optimal marketing effort e^* are therefore evident. □

3.2. Multiplicative demand model

Let the demand function (2) be defined in multiplicative form, $D_M(e, \xi_m) = d(e)\xi_m$. Considering model (1), the multiplicative demand form, and defining the stocking factor $z \in \mathbb{R}$ as $z = \frac{x}{d(e)}$ (if $x = 0$ then $e = 0$ but $d(0) > 0$), the expected profit can be rewritten as

$$\Pi(e, z) = \Psi(e) - L(e, z) = d(e)[p - c - l(z)] - C(e), \tag{6}$$

where $L(e, z) = d(e)l(z)$ is the expected loss that occurs as a result of the presence of uncertainty and $p-c-l(z)$ denotes the so-called per-unit expected benefit (margin minus expected loss), $\Psi(e)$ and $l(z)$ are defined in section 3.1, $d(e)\Lambda(z)$ denotes expected leftovers and $d(e)\Theta(z)$ expected shortages. Thus, the following assumption is reasonable [1]:

Assumption 1. The per-unit expected benefit is positive; that is, $p - c - l(z^*) > 0$.

From expression (6), we can see that the decision on optimal stocking factor z^* is not influenced by marketing effort e, as in the additive demand model; therefore, it is given by expression (5). In contrast, the decision on optimal marketing effort e^* is not made independently of z unlike in the additive demand model and is given by

$$[p - c - l(z^*)] \frac{\mathrm{d}d(e)}{\mathrm{d}e} - \frac{\mathrm{d}C(e)}{\mathrm{d}e} = 0. \tag{7}$$

The definition of the stocking factor then provides the optimal ordering amount as $x^* = z^* d(e^*)$.

Example 3 (Optimal marketing effort for the linear demand case). Let the functions $d(e)$ and $C(e)$ be given by the same expressions as in Example1.Then, by substituting the functions into condition (7), we can obtain the optimal marketing effort, $e^* = \frac{[p-c-l(z^*)]b}{\mu}$.

Example 4 (Optimal marketing effort for the concave demand case). Let Assumption 1 hold true and let the functions $d(e)$ and $C(e)$ be given by the same expressions as in Example 2.Then, substituting $d(e)$ and $C(e)$into condition (4), we can obtain the optimal marketing effort, $e^* = \left\{\frac{[p-c-l(z^*)]ab}{\mu}\right\}^{\frac{1}{2-b}}$.

3.2.1. Unification of findings from the systematic review

Theorem 2. *Let functions $d(e)$ and $C(e)$ satisfy multiplicative model assumptions. Assumption 1 holds, and let the demand-effort response function $d(e)$ be increasing and concave, and the cost of marketing effort $C(e)$ be increasing and convex. Then, the expected profit $\Pi(e, z^*)$ is concave in e, and the globally optimal marketing effort e^* is unique. If it is positive, then it is given by the solution of (7).*

Proof. This is analogous to the proof of Theorem 1. □

Let the marketing effort elasticity of the demand-effort response function $d(e)$ be defined as $\frac{e}{d(e)} \cdot \frac{\mathrm{d}d(e)}{\mathrm{d}e}$. Then, using the following two assumptions, Theorem 2 can be generalized to Theorem 3, see [30].

Assumption 2. (i) The demand-effort response function $d(e)$ is strictly increasing and has non-increasing effort elasticity, and (ii) the cost of marketing effort $C(e)$ is strictly increasing and has non-decreasing effort elasticity.

Remark. A discussion of two cases for the marketing effort cost $C(e)$ elasticity is provided in [30], namely of the cases of elasticity smaller than1 and greater than or equal to 1.

Assumption 3. The random factor ξ_m has an increasing generalized failure rate (IGFR) within a specified range, for example, $[A_m, B_m]$.

Remark. The IGFR assumption is satisfied by many commonly used distributions, such as uniform, normal, exponential, gamma, and Weibull distributions (with certain restrictions on the shape parameter). [30]

Theorem 3 ([30])**.** *Under Assumptions 1, 2 and 3, the expected profit $\Pi(e, z^*)$ is quasi-concave in e and there exists aunique optimal marketing effort e^*.*

Proof. See [30]. □

3.2.2. An extension to the S-Shaped demand-effort response function

In the following, we deal with an S-shaped function, $d(e)$. We must relate this to the function $C(e)$. Therefore, we formulate the following two assumptions. The first concerns the relationship between the functions for "big" arguments e; the latter will concern the relationship of the functions for "small" arguments e. This distinction results from the different growth rates of the S-shaped function for edge arguments.

Assumption 4. If $d(e)$ is increasing and an S-shaped response function, $C(e)$ is the cost of marketing effort, and z^* denotes the optimal stocking factor (if it exists), then there is an e_c such that the condition $\frac{\mathrm{d}d(e)}{\mathrm{d}e}/\frac{\mathrm{d}C(e)}{\mathrm{d}e} < 1/[p-c-l(z^*)]$ is satisfied for all $e > e_c$.

Remark. (i) Assumption 4 represents the condition that $d(e)$ grows slower than $C(e)$ by factor $1/[p-c-l(z^*)]$ for $e > e_c$.
(ii) If an S-shaped response function $d(e)$ is constrained by a pair of horizontal asymptotes as $x \to \pm\infty$ (which is the usual demand for S-shaped functions), which implies $\lim_{e\to e_f^-} \frac{\mathrm{d}d(e)}{\mathrm{d}e} = 0$ for some e_f or $\lim_{e\to\infty} \frac{\mathrm{d}d(e)}{\mathrm{d}e} = 0$, then Assumption4 is satisfied.

Lemma 1. Let functions $d(e)$ and $C(e)$ satisfy multiplicative model assumptions, $d(e)$ be an increasing S-shaped function, and let Assumptions 1 and 4 hold. If the cost of marketing effort $C(e)$ is an increasing function, then there is a$\tilde{e}$ such that the expected profit $\Pi(e, z^*)$ is decreasing for all $e > \tilde{e}$.

Proof. The lemma will be proved by contradiction. Suppose that the expected profit $\Pi(e, z^*)$ is non-decreasing for all$e$. Then, its first derivative is non-negative for all$e$:

$$\frac{\mathrm{d}\Pi(e,z^*)}{\mathrm{d}e} = [p-c-l(z^*)]\frac{\mathrm{d}d(e)}{\mathrm{d}e} - \frac{\mathrm{d}C(e)}{\mathrm{d}e} \geq 0 \quad \text{for all } e. \tag{8}$$

Because$p-c-l(z^*) > 0$according to Assumption1 and $\frac{\mathrm{d}C(e)}{\mathrm{d}e} > 0$ (because $C(e)$ is anincreasing function), it follows that

$$\frac{\mathrm{d}d(e)}{\mathrm{d}e}/\frac{\mathrm{d}C(e)}{\mathrm{d}e} \geq 1/[p-c-l(z^*)] \quad \text{for all } e. \tag{9}$$

However, this contradicts assumption4. □

Assumption 5. Let the function $d(e)$ be increasing and S-shaped with apoint of inflection $e_I > 0$ and z^* denote the optimal stocking factor (if it exists). Furthermore, let the functions $d(e)$ and $C(e)$ satisfy the condition $\frac{\mathrm{d}d(e)}{\mathrm{d}e}/\frac{\mathrm{d}C(e)}{\mathrm{d}e} > 1/[p-c-l(z^*)]$ for all $e \in (0, e_I)$.

Remark. Assumption 5 represents the condition that $d(e)$ grows on the interval $(0, e_I)$ faster than $C(e)$ by factor $1/[p-c-l(z^*)]$.

Theorem 4. *Let functions $d(e)$ and $C(e)$ satisfy multiplicative model assumptions, and assumptions 1, 4 and 5 hold. If the demand-effort response function $d(e)$ is increasing and S-shaped and the cost of marketing effort $C(e)$ is increasing and convex, then the expected profit $\Pi(e, z^*)$ is strictly quasi-concave in e, and the globally optimal marketing effort is unique and is given by (7).*

Proof. Expression6 and Assumption5 implythat:

$$\frac{\mathrm{d}\Pi(e,z^*)}{\mathrm{d}e} = [p-c-l(z^*)]\frac{\mathrm{d}d(e)}{\mathrm{d}e} - \frac{\mathrm{d}C(e)}{\mathrm{d}e} > \frac{\mathrm{d}C(e)}{\mathrm{d}e} - \frac{\mathrm{d}C(e)}{\mathrm{d}e} = 0 \quad \text{for all } e \in (0, e_I). \tag{10}$$

Hence, the expected profit function $\Pi(e, z^*)$ is strictly increasing in interval $(0, e_I)$. Because Theorem2 holds true for $e > e_I$, the expected profit $\Pi(e, z^*)$ is strictly quasi-concave in e. Furthermore, from Lemma1, there is an$\tilde{e}$ such that the expected profit $\Pi(e, z^*)$decreases for all $e > \tilde{e}$. Thus, $\Pi(e, z^*)$ changes its monotonicity from increasing to decreasing, which implies the existence of a globally optimal marketing effort, which follows from the quasi-concavity; it is clearly given by (7). □

The theorems and assumptions described in this section are summarized in Table4.

The remainder of Section 3.2 is devoted to several examples of the S-shaped demand-effort response function and its effect on the expected profit function.

Example 5 (Profit function for the S-shaped demand case). If $p-c-l(z^*) = 1$, i.e.,$\Pi(e, z^*) = d(e) - C(e)$, then functions $d(e) = 2\exp(4e-4)/(1+\exp(4e-4))$ and $C(e) = 0.4e^2$ (satisfying assumptions of Theorem4) lead to the expected profit $\Pi(e, z^*)$ illustrated in Figure 2.

Four examples of the demand-effort response function $d(e)$, marketing effort cost $C(e)$ and related expected profit function $\Pi(e, z^*)$, where either Assumption 4 or Assumption 5 are not met, are presented in Table 5. Figures 3–6 demonstrate these various cases which can occur if some of Assumptions4 and5 is not satisfied. The relationship between the cases and the aforementioned assumptions is described in Table5.

Table 4: Summary of the theorems and assumptions

Theorem	Form		Function properties		Assumptions					Expected profit
	A	M	$d(e)$	$C(e)$	1	2	3	4	5	$\Pi(e, z^*)$
1	✓		concave	convex	✓*					concave
2		✓	concave	convex	✓					concave
3		✓	non-increasing elasticity	non-decreasing elasticity	✓	✓	✓			quasi-concave
4		✓	S-shaped	convex	✓			✓	✓	quasi-concave

Note: *for the additive demand case, the equivalent of Assumption 1 is $p - c > 0$

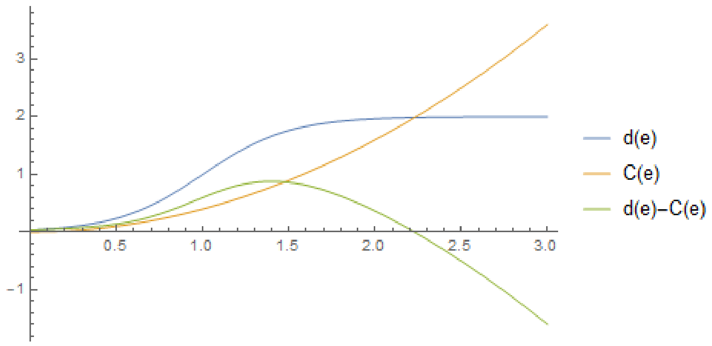

Figure 2: Illustration of Example 5

Table 5: Some cases of $d(e)$ or $C(e)$ which do not satisfy Assumptions 4 and 5

Example #	$d(e)$	$C(e)$	Figure #	Assumption 4	Assumption 5
Example 6	$\frac{e^2}{2e+1} + \arctan(e-3) + 1.4$	$\frac{e^2}{5e+1}$	Figure 3	×	✓
Example 7	$\frac{4\exp(4e-7)}{1+\exp(4e-7)}$	$0.4e^2$	Figure 4	✓	×
Example 8	$\frac{2\exp(4e-4)}{1+\exp(4e-4)}$	$1.1(e+0.1)^2$	Figure 5	✓	×
Example 9	$\arctan(9e-5) + 1.5$	$7e^3$	Figure 6	✓	×

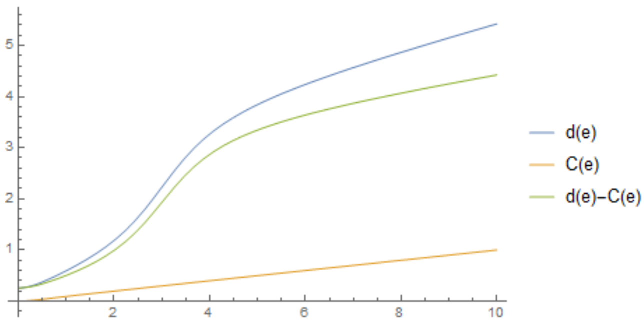

Figure 3: Illustration of Example 6

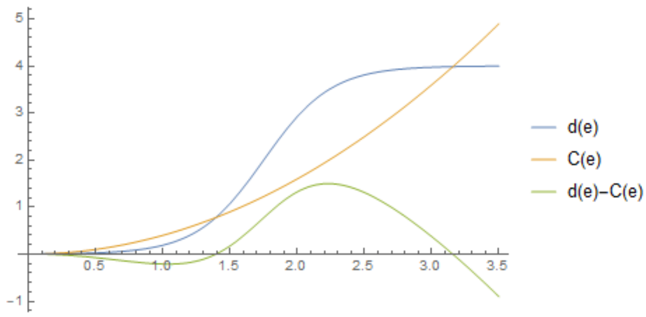

Figure 4: Illustration of Example 7

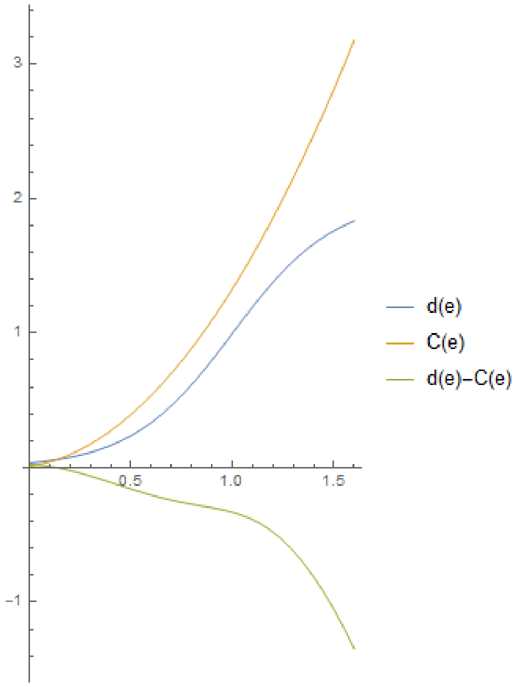

Figure 5: Illustration of Example 8

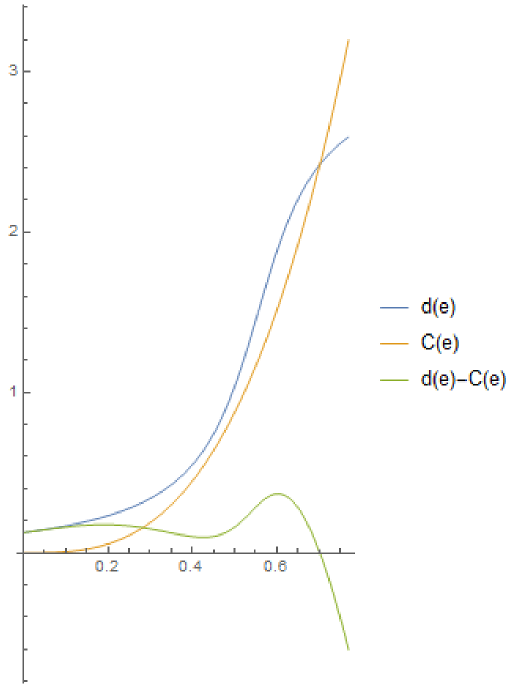

Figure 6: Illustration of Example 9

Remark. Note that the expected profit $\Pi(e, z^*)$ can have none or more than one local extreme if Assumption 5 is not satisfied. This is illustrated in figures 5–6.

3.3. Sensitivity analysis: Impact of parameters changes on the optimal marketing effort

In this section, the influence of selling price p and buying cost c on the optimal marketing effort is analyzed.

3.3.1. Additive demand form

Realizing that the optimal marketing effort $e^* > 0$ is determined by Equation(4), it is obvious that increasing the selling price p to $\widetilde{p}$ leads to a new optimal marketing effort $\tilde{e}$ which (if it exists and is positive) is determined by the analogous equation:

$$(\widetilde{p} - c)\frac{\mathrm{d}d(\tilde{e})}{\mathrm{d}e} - \frac{\mathrm{d}C(\tilde{e})}{\mathrm{d}e} = 0. \tag{11}$$

Regarding the mutual relationship between e^* and $\tilde{e}$, one would expect that $\widetilde{p} > p$ implies $\tilde{e} > e^*$. The following theorem describes this more precisely.

Theorem 5. *Let functions $d(e)$ and $C(e)$ satisfy the additive model assumptions, $p - c > 0$, $C(e)$ be increasing and convex, $d(e)$ be increasing and concave or S-shaped. Furthermore, let the optimal marketing effort e^* be determined by Equation(4), and the optimal marketing effort $\tilde{e}$ (related to $\widetilde{p} > p$) be determined by Equation(11). Then, $\tilde{e} > e^*$.*

Proof. Because e^* represents a maximum of $\Psi(e) = (p - c)d(e) - C(e)$ (and because of all the assumptions of the theorem), it follows that

$$\frac{\mathrm{d}\Psi(e^*)}{\mathrm{d}e} = 0 \quad \text{and} \quad \frac{\mathrm{d}\Psi(e)}{\mathrm{d}e} < 0 \quad \text{for all} \quad e > e^*. \tag{12}$$

Hence,

$$p - c = \frac{\mathrm{d}C(e^*)}{\mathrm{d}e} \Big/ \frac{\mathrm{d}d(e^*)}{\mathrm{d}e} \quad \text{and} \quad p - c < \frac{\mathrm{d}C(e)}{\mathrm{d}e} \Big/ \frac{\mathrm{d}d(e)}{\mathrm{d}e} \quad \text{for all} \quad e > e^*. \tag{13}$$

Defining $\varphi(e) = \frac{\mathrm{d}C(e)}{\mathrm{d}e} / \frac{\mathrm{d}d(e)}{\mathrm{d}e}$ leads to the following (abbreviated) expressions:

$$p - c = \varphi(e^*) \quad \text{and} \quad p - c < \varphi(e) \quad \text{for all} \quad e > e^*. \tag{14}$$

Similarly, because $\tilde{e}$ represents the maximum of $\widetilde{\Psi}(e) = (\widetilde{p} - c)d(e) - C(e)$, the following holds true:

$$\frac{\mathrm{d}\widetilde{\Psi}(\tilde{e})}{\mathrm{d}e} = 0 \quad \text{and} \quad \frac{\mathrm{d}\widetilde{\Psi}(e)}{\mathrm{d}e} < 0 \quad \text{for all} \quad e > \tilde{e}. \tag{15}$$

Analogous to the previous part, it can be rewritten as

$$\widetilde{p} - c = \varphi(\tilde{e}) < \varphi(e) \quad \text{for all} \quad e > \tilde{e}. \tag{16}$$

Assumption $e^* \geq \tilde{e}$ implies $p - c < \widetilde{p} - c \leq \varphi(e^*) = p - c$ which is contradictory. Thus, $\tilde{e} > e^*$. □

Remark. Note that existence of e^* (determined by Equation(4)) does not imply existence of $\tilde{e}$ (determined by Equation(11)). See, e.g., Figure 7.

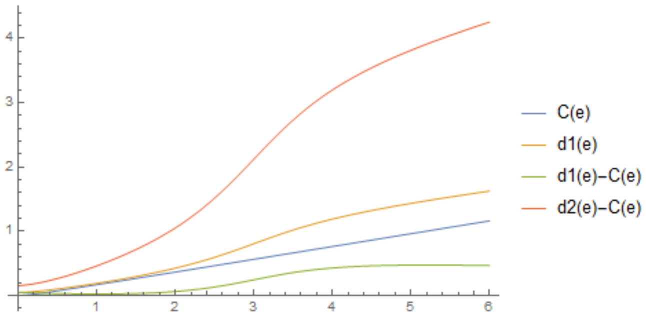

Figure 7: Illustration of the remark above: existence of e^* does not imply existence of $\tilde{e}$; $C(e) = \frac{e^2}{5e+1}$, $d_1(e) = 0.3\left(\frac{e^2}{2e+1} + \arctan(e-3) + 1.4\right)$, $d_2(e) = \frac{e^2}{2e+1} + \arctan(e-3) + 1.4$

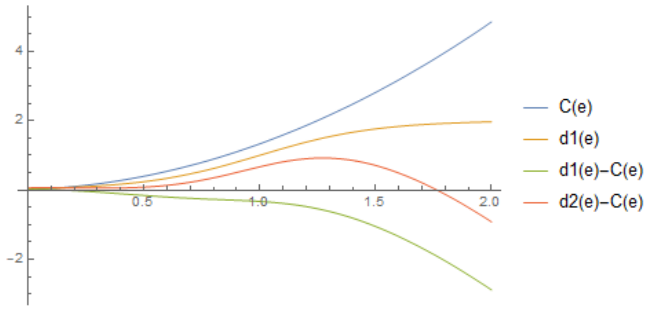

Figure 8: Illustration of the remark below: non-existence of e^* does not imply non-existence of $\tilde{e}$; $C(e) = 1.1(e+0.1)^2$, $d_1(e) = \frac{2\exp(4e-4)}{1+\exp(4e-4)}$, $d_2(e) = \frac{4\exp(4e-4)}{1+\exp(4e-4)}$

Remark. Note that non-existence of e^* (determined by Equation(4)) does not imply non-existence of $\tilde{e}$ (determined by Equation(11)). See, e.g., Figure 8.

Effect of increasing the value of c (while $c \leq p$) on decreasing the optimal marketing effort is demonstrated in the next theorem (which follows Theorem5).

Theorem 6. *Let functions $d(e)$ and $C(e)$ satisfy the additive model assumptions, $p - c > 0$, $C(e)$ be increasing and convex, $d(e)$ be increasing and concave or S-shaped. Furthermore, let the optimal marketing effort e^* be determined by Equation(4), and the optimal marketing effort $\tilde{e}$ (related to $\tilde{c} > c$ where $p - \tilde{c} > 0$) be determined by Equation(11). Then, $\tilde{e} < e^*$.*

Remark. Considering the partial derivative of the expected profit equation with respect to the marketing effort e, e.g., as given by the expression (4), similar results can easily be achieved with the help of the following inequalities

$$\frac{\partial\left(\frac{\partial\Pi(e,z^*)}{\partial e}\right)}{\partial p} = \frac{\left[(p-c)\frac{\mathrm{d}d(e)}{\mathrm{d}e} - \frac{\mathrm{d}C(e)}{\mathrm{d}e}\right]}{\partial p} = \frac{\mathrm{d}d(e)}{\mathrm{d}e} > 0$$

and

$$\frac{\partial\left(\frac{\partial\Pi(e,z^*)}{\partial e}\right)}{\partial c} = \frac{\left[(p-c)\frac{\mathrm{d}d(e)}{\mathrm{d}e} - \frac{\mathrm{d}C(e)}{\mathrm{d}e}\right]}{\partial c} = -\frac{\mathrm{d}d(e)}{\mathrm{d}e} < 0.$$

In summary, it was shown (under the assumptions described above) that increasing the selling price p leads to an increase in the optimal marketing effort e^*, whereas increasing the buying cost c leads to a decrease in e^*. Regarding the other parameters, it can easily be verified that the unit shortage penalty cost s and unit salvage value v have no impact on the optimal marketing effort e^*.

3.3.2. Multiplicative demand form

Consideration of the partial derivative of the expected profit with respect to the marketing effort, for example, given by the expression (7), leads to a form that also depends on the expected loss $l(z^*)$ which depends on both parameters, p and c. Therefore, for the multiplicative demand form, a particular distribution must be applied to obtain a similar observation for the optimal marketing effort e^*.

Uniform distribution. The following examples are defined for the multiplicative case. Let the random variable ξ_m be uniformly distributed, that is, $\xi_m \sim U(A_m, B_m)$. Subsequently, from (5), we obtain $z^* = A_m + \frac{(p+s-c)(B_m-A_m)}{p+s-v}$. Substituting z^* into the expected loss function $l(z)$ we obtain $l(z^*) = (z^* - A_m)\frac{c-v}{2} = \frac{B_m-A_m}{2}(c-v)\frac{p+s-c}{p+s-v}$. Using the obtained $l(z^*)$, condition $p - c - l(z^*) > 0$ from Assumption 1 is converted to $p - c - \frac{B_m-A_m}{2}(c-v)\frac{p+s-c}{p+s-v} > 0$.

Let us take the first derivative of the expression given by (7) with respect to e and substitute z^* (and $l(z^*)$ respectively) for the uniform distribution. Then, by taking the derivative with respect to p, we obtain:

$$\frac{\partial\left(\frac{\partial \Pi(e,z^*)}{\partial e}\right)}{\partial p} = \frac{\mathrm{d}d(e)}{\mathrm{d}e}\left[1 - \frac{B_m - A_m}{2} \cdot \frac{(c-v)^2}{(p+s-v)^2}\right] > 0.$$

Therefore, the optimal marketing effort is strictly increasing in selling price p.

Using the same procedure, but with respect to c, we obtain

$$\frac{\partial\left(\frac{\partial \Pi(e,z^*)}{\partial e}\right)}{\partial c} = d(e)\left[\frac{B_m - A_m}{2} \cdot \frac{2c-p-s-v}{p+s-v} - 1\right] < 0,$$

which means that the optimal marketing effort is strictly decreasing in buying cost c.

Similarly, we consider the derivatives with respect to s and v:

$$\frac{\partial\left(\frac{\partial \Pi(e,z^*)}{\partial e}\right)}{\partial s} = \frac{\mathrm{d}d(e)}{\mathrm{d}e}\left[-\frac{B_m - A_m}{2} \cdot \frac{(c-v)^2}{(p+s-v)^2}\right] < 0$$

and

$$\frac{\partial\left(\frac{\partial \Pi(e,z^*)}{\partial e}\right)}{\partial v} = \frac{\mathrm{d}d(e)}{\mathrm{d}e}\left[\frac{B_m - A_m}{2} \cdot \frac{1}{(p+s-v)^2}\right] > 0.$$

3.3.3. Summary of both demand forms

Table 6 summarizes the above findings under the studied circumstances for both the additive and multiplicative demand cases. For the additive form, the findings hold in general (for the given assumptions, see Section 3.3.1), and the multiplicative case is illustrated for the uniform distribution of the random variable (see Section 3.3.2).

Table 6: Impact of parameters changes on the optimal marketing effort

Parameter	Additive demand form	Multiplicative demand form	
	Conclusion	Additional assumption	Conclusion
p	$\Pi(e, z^*)$ is strictly supermodular in (e, p)	Uniform distribution	$\Pi(e, z^*)$ is strictly supermodular in (e, p)
c	$\Pi(e, z^*)$ is strictly submodular in (e, c)	Uniform distribution	$\Pi(e, z^*)$ is strictly submodular in (e, c)
s	s has no effect on the optimal e	Uniform distribution	$\Pi(e, z^*)$ is strictly submodular in (e, s)
v	v has no effect on the optimal e	Uniform distribution	$\Pi(e, z^*)$ is strictly supermodular in (e, v)

3.4. Discussion: Managerial insights and applicability

In light of these observations, a few managerial implications can be derived from the results. At the general level, the findings lead to a better understanding of how inventory and production policies should be structured when the objective is severity-based and how this should be done in connection with the marketing effort decision. Using the models investigated, inventory managers facing uncertain demand can utilize marketing efforts to reduce their marketing expenditures and increase expected profits. According to the particular market (e.g., based on some historical or actual information), they can take into account market behavior with the help of the marketing response function and so better predict the demand and react to the actual state. As seen from the results, the optimal marketing effort decision strongly relates to the choice of (e.g., S-shaped related) market; it can be determined relatively simply by applying the obtained results and knowledge. Similarly, the effects of various parameters (buying cost, selling price, shortage cost, and salvage value) are evident and provide valuable insights for inventory managers. While the optimal marketing effort increases with the increase in the selling price and decreases with the increase in the buying cost in general (regardless of the random demand case), there are different results for the shortage penalty cost and salvage value: both parameters do not affect the optimal effort for the additive case; a higher shortage penalty cost leads to a lower optimal effort, and a higher salvage value leads to higher optimal effort in the multiplicative demand form.

The application of this theory allows the incorporation of other parameters and variables (e.g., other market-related response functions, variables such as pricing, or various manager risk preferences) into the formulation of the newsvendor model. A very important advantage of the model proposed in this study is its ability to determine the optimal inventory based on measurable market variables. While maintaining high levels of inventory can be expensive, and retaining low inventory levels can negatively impact customer service, a middle ground can be found by building carefully planned inventory levels.

The newsvendor-like models can be applied in various areas where retailers, manufacturers, managers, and decision-makers have to decide on optimal inventory levels. However, in connection with the marketing effort decision, the developed model and obtained results can be applied and utilized in problems where the retailer decides not only on the inventory level but also affects the market (customers' demand). This single-period model is regularly used when selling and promoting a non-perishable product to maximize expected profit. However, our results can also be applied in other non-traditional application areas where the marketing decision can go along with the inventory decision, such as the pharmaceutical industry [4] and other reverse logistics-based application areas [6, 5].

4. Conclusions

This study deals with the widely used newsvendor problem with marketing efforts. In recent decades, many different formulations of the newsvendor problem and marketing efforts have been defined. This study uses a systematic review as a suitable tool to review, summarize, and generalize a wide set of formulations and results. According to the findings observed during the systematic review, the newsvendor problem with marketing efforts was formulated with generalized assumptions.

Two commonly used forms of random demand have been analyzed: additive and multiplicative. For both cases, an illustrative notation defining the riskless profit, expected loss functions, expected leftovers, and shortages is used. This allows us to obtain results concerning the optimal decisions relatively simply (namely, results concerning the marketing effort, optimal ordering quantity, and stocking factor). The optimal stocking factor leads to the same expression for both cases and is influenced by the uncertainty considered. On the other hand, the marketing effort is not influenced by the uncertainty and the expected loss function in the additive demand form; thus, it corresponds to the riskless profit, unlike in the multiplicative demand form, where the expected loss function appears as a result of uncertainty. For both demand forms, illustrative examples of the typically used demand-effort response functions and the cost of marketing effort functions are provided, as well as theorems guaranteeing the uniqueness of the optimal marketing effort. Finally, we show that the optimal marketing effort is strictly increasing the selling price and strictly decreases the buying cost for both demand forms. Therefore, under the studied circumstances, an increase in the unit profit margin, that is, the difference between the selling price and buying cost, leads to a higher optimal marketing effort. However, the shortage penalty cost and the salvage value have no effect on the optimal marketing effort for the additive demand form. The optimal marketing effort strictly decreases the shortage penalty cost. and strictly increases salvage value.

Altogether, this paper presents an efficient tool to review and examine existing results in the field of inventory optimization and provides an illustrative methodology to extend recent results relatively simply. Further work may span extensions on, for example, pricing decisions, analysis of variance, and decision-dependent randomness.

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Appendix A. Additional findings of the systematic review

Table A.1: Detailed examples of the demand-effort response function $d(e)$ and marketing effort cost $C(e)$

Article	Form		Demand function		Marketing effort		
	A	M	$d(e)$	Parameters	$C(e)$	Properties	Parameters
[28]	✓		$a+e$	$a>0$	αe^2	$C(0)=0, C'(e)>0$, twice differentiable	$\alpha>0$
[29]	✓		$a+be$	$a,b>0$	αe^2	$C(0)=0, C'(e)>0, C''(e)>0$	$\alpha>0$
[20]	✓		$a+be$	$a,b>0$	αe^2	–	–
[2]	✓		be	$b>0$	αe^2	–	$\alpha>0$
[22]	✓		$A: a+be$	$a,b>0$	$G^{-1}((1-\gamma)e)$	$C'(e)>0, C''(e)>0$	$\gamma>0, G^{-1}$
		✓	ae^{δ}	$\delta\in(0,1)$			
[11]	✓		concave		αe^2	$C'(e)>0, C''(e)>0$, $C(0)=0, C(\infty)=\infty$	$g>0$
[24]	✓	✓	increasing			$C''(e)>0$	
[31]	✓	✓	$\frac{\tau e}{1+e}$	$\tau>0$	αe^{β}	–	$\alpha,\beta>0$
		✓	ae^{δ}	$\delta\in(0,1)$			increasing and convex
[15]		✓	ae^{δ}	$a>0, \delta\in(0,1]$	αe^2	$C'(e)>0$, $C''(e)>0$	–
[27]		✓	e		$\alpha(e-1)^2\xi^{\delta}$	$C(e,\xi)$: convex, increasing and continuously differentiable in e for any $e\geq 1$, continuous in ξ for any $e\geq 1$, $C(1,\xi)=0$	$\alpha>0$, $\delta\in(-\infty,\infty)$
[7]		✓	e		αe^{β}	$C'(e)>0, C''(e)>0$, $C(0)=0, C(\infty)=\infty$	$\beta>1$
[19]		✓	e		αe^2	$C(0)=0, C'(e)>0, C''(0)>0$	$\alpha>0$
[18]		✓	be	$b>0$	αe^2	$C(0)=0, C'(e)>0$, $C''(e)>0$	–
[23]		✓	e		αe^2	$C(0)=0, C'(e)>0, C''(e)>0$	$\alpha>0$
[9]		✓	$1+be$		αe^2	$C'(e)>0, C''(e)>0, C(0)=0$	–
[17]		✓	$a[1-bexp(ce)]$	–	e^2	$C(0)=0, C'(0)=0, C'(e)\geq 0, C''(e)>0$	–
[32]			$a+be$	$b\geq 0$	αe^2	–	$\alpha>0$
[8]			–	mean μ_i	ηe	–	$\eta>0$